Contribution of a mathematical model of specifications of a part to their coherence analysis

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Abstract : This paper presents a method to check the validity of the geometric specifications of a given part. We use a substitute part constituted with exact surfaces and we assemble this part on another which represents both the specified datum frame and the location of the tolerance zone. This model permits to know if some part features are over constrained or if the specification is not sufficient to define the location of the tolerance zone.
Keywords : tolerance modeling, tolerance analysis, tolerance coherence, deviation area

1. INTRODUCTION

The analysis of the tolerancing validity of the parts of a mechanism is a research field developed in numerous papers [Clément et al., 1991], [Requicha, 1983], [Turner, 1990]. The usually used methods suppose that the tolerancing of the parts of the mechanism is known and correct. In this paper, we propose a method which permits to check the coherence of the tolerancing of a part by a formal mathematical analysis of every geometric specification on the part. This analysis is done by the assembly of the substitute part of the actual part on a true part which represents the geometric specification. This analysis permits to know and to quantify the specification lacks and the over-constrained features on the part. This study completes a first study [Leveaux 92] which gives both a syntax analysis of the geometrical specification and a validation of the specification based on the pertinence of their inspection.

2. GEOMETRIC TOLERANCING PRINCIPLES

2.1 Without options: MMC (M), LMC (L), projected tolerance zone P:

ISO standard permits to point out either actual features or true features derived from the true geometric counterpart of features according to constraints.

So actual features are either the toleranced feature pointed out by an arrow or the datum features pointed out by a triangle. These features may be established by datum targets and be a combination of features, or be an area of the feature. The toleranced feature may also be a derived median plane of two actual parallel planes, or the derived axis of an actual cylinder or cone.

The true features are:

- Specified (single or common) datum features, and specified datum frames. These are mainly points, lines and planes which may be geometrically constrained in orientation or in position. Single datum or common datum will be derived from the actual features accordingly to the same association criteria. The specified datum of a datum frame will be associated one after the other, in a sequential and arranged way.
- Tolerance zones which may be constrained in orientation or in position relatively to the specified datum.

The toleranced features have to be included in the described tolerance zones. Each specification needs to be satisfied regardless of the other specifications.

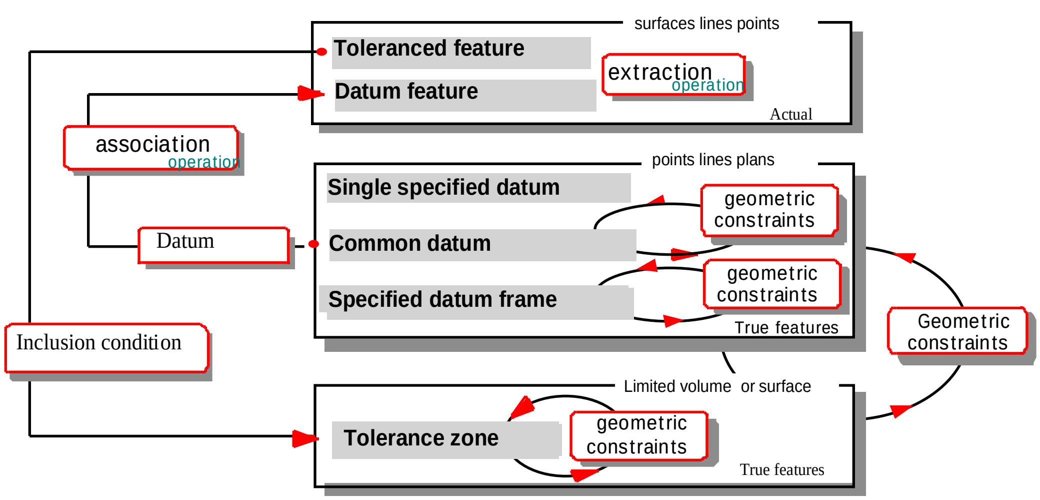


Figure 1 *: ISO tolerance specification reading (without any option)*

2.2 With options: MMC (M), LMC (L), projected tolerance zone P

In the case a least or a maximum material condition option is used, the signification of the specification is changed. ISO standard presents the virtual condition for the toleranced feature and the maximum (or minimum) material size for the datum feature simulator, these two states must not be crossed by the material (or being in the material). The virtual state is defined by the true envelope which dimension is given by the cumulated effect of the maximum material dimension (or least material dimension) and of the geometric tolerance.

2.3 Specification expression

In order to illustrate our approach, we will use the part presented in figure 2 as an example. This part, coming from the industry and toleranced by a group of expert, is reproduced here in a stylized way.

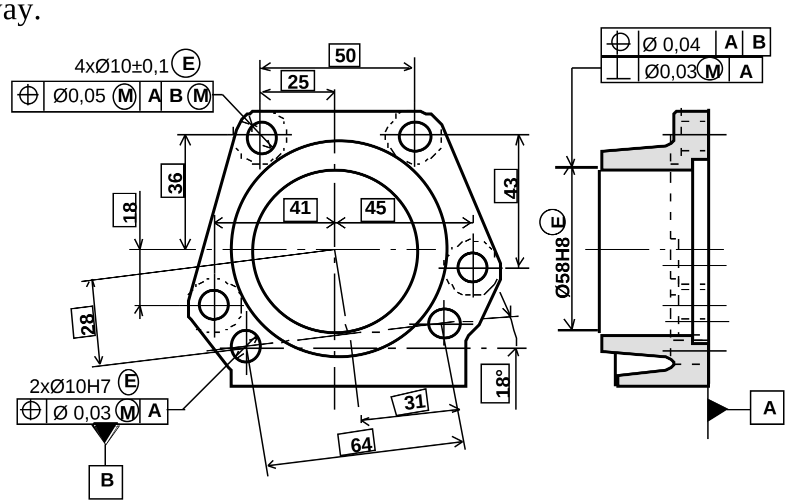


Figure 2 *: definition drawing of a cover*

Specification	*Toleranced feature*	*Datum feature*	*Datum and maximum material condition datum*	*Tolerance zone or virtual condition size*
⊥ | Ø0,03 Ⓜ | A	Actual bore (Ø58)	Actual feature A	True plane A associated with feature A	Cylinder Ø57.97 normal to planeA
⌖ | Ø0,05 Ⓜ | A | B Ⓜ	Four Actual bores (Ø10±0,1)	Actual feature A and two actual bores (Ø10H7)	True plane A associated with feature A and two cylinders Ø10, normal to A, in a theoretical position.	Four cylinders Ø9,85, normal to plane A, in a theoretical position.

Table I *: Studied tolerance description examples*

3. SPECIFICATION MODEL BY ASSEMBLY CALCULATION

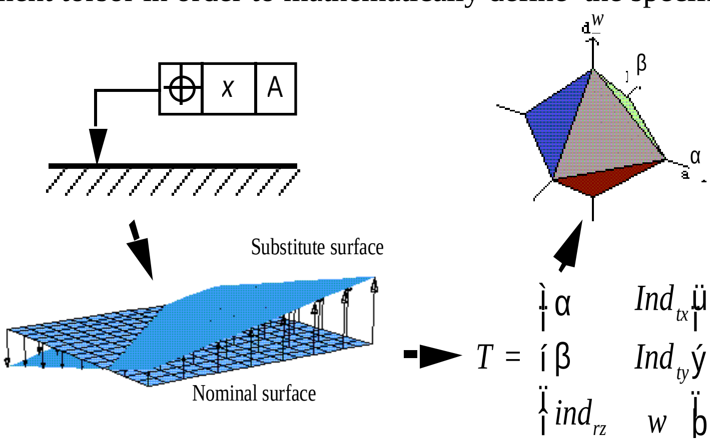
In order to define a mathematical model for the geometrical specifications of a part, we will apply a calculation method of three-dimensional chain of dimension to a part and its specification. We will first describe why this transposition is possible and then give the principles and the modes of enforcement.

3.1 Identity between the parameters used in three-dimensional metrology, the three-dimensional chain of dimension and the specifications

The small displacement torsors permits a representation of the deviations between a nominal (or theoretical) geometry and a substitute geometry. This substitute geometry is already used in three-dimensional metrology to identify actual surfaces and build some specified reference frames.

On a complementary way, numerous model of three-dimensional chain of dimensions are based on a similar representation. The small displacement torsor is the tool which permits to express the deviation between the nominal representation and the substitute representation of the surfaces, the gap between two surfaces belonging to different parts and the position of the part relative to their nominal position.

The same tool can also be used to mathematically describe the tolerances. Then the components of the torsor provide a representation of the geometrical deviation of the substitute feature in a different space: the space of the small torsor displacement of the toleranced feature relative to the nominal feature cf. figure 1. The limits corresponding to the tolerance zone define then another expression of the specifications: the specified deviation area. A systematic methodology has then to be given to calculate the constraints on the small displacement torsor in order to mathematically define the specifications.



$$T = \begin{Bmatrix} \alpha & Ind_{tx} \\ \beta & Ind_{ty} \\ ind_{rz} & w \end{Bmatrix}$$

***Figure 3** : changed space used for the model*

3.2 General principle of the calculation of the specified deviation area

An equivalence can be provided between a geometrical deviation between some features of a part and a functional condition relative to a mechanism. Indeed, the substitute part will verify the specification if it can be assembled in true parts which represent the tolerance zone of each specification. The specified datum frame of each specification defines the assembly conditions between the substitute and the true model of the part. The tolerance zone is then defined as a condition to be verified by the substitute feature relative to the nominal position.

This equivalence between the functional condition expressed on a mechanism and the specification relative to features of a part permits to use the results of the model study of the three-dimensional chain of dimension presented during the last CIRP seminar on Tolerancing [BALLOT 97]. The geometric laws of comportment expressed on the feature of a part give the relations to verify between the substitute and the nominal features. The deviations calculated with the behavior laws give the relation to be verified by the substitute surfaces to verify the specification.

The expression of these deviations on a sampling of the tolerance zone defines then the mathematical constraints which represent the specification in the space of the small displacement torsor parameters.

Table 2, presented below, resumes the correspondence between the standard geometric specification feature and the features used in the model that we will describe.

Standard feature	*Proposed model*
Toleranced feature	Substitute feature
Datum feature	Substitute feature
Specified datum	Theoretical feature or their assembly
Tolerance zone	Geometric zone where the deviation calculated with the behavior law is sampled
Options Ⓔ	Always taken into account by the model
Ⓜ Ⓛ	Like the relations are expressed between the features, they correspond to the use of the least or maximum material condition for lack.

***Table II** : correspondence between the standard and the model features*

The systematic calculation of the deviation area is based on the use of the model of calculation of the laws of geometrical behavior presented during the last CIRP seminar on tolerancing. That is why we will describe above all his application in this new utilisation context: the assembly of an substitute part in a set of true parts which represents the specified datum frames and the tolerance zones.

3.3 The small displacement torsor

Like numerous other works in this field of research, we use the small displacement torsor to characterize the deviations between the substitued model and the nominal model [Bourdet et al., 1996]. Three kinds of deviation are represented by different torsor:

The deviation between a substitute feature and its nominal definition is characterized with a deviation torsor. The form of this torsor is deduced of the geometrical feature type (plane, cylinder, etc).

The gap between two features belonging to two different parts is characterized by a gap torsor. For the presented application, the gap deviation is defined between the substitute surface of the part and a true feature representing the tolerance zone and placed relatively to the datum frame. The form of this torsor is determined by the (same) type of the two surfaces.

The small displacement torsor which is necessary to place the part on each of the specified datum frame is characterized by a part torsor.

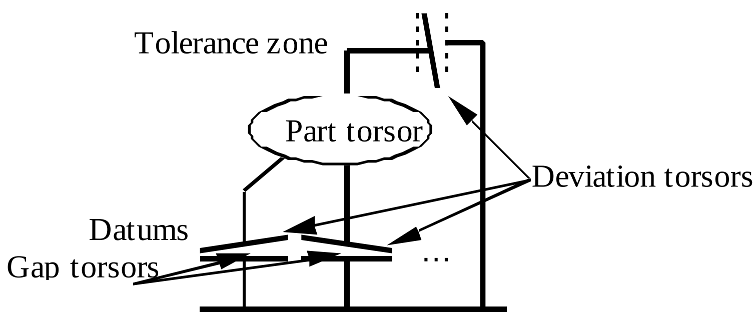


***Figure 4** : general description graph of a specification*

3.4 Specified datum frame: assembly between a substitute model and the true model

We will successively examine the case of a single datum and the case of a datum frame.

The positioning of a tolerance zone relatively to a simple reference represents an assembly between the substitute feature (which is the toleranced feature) and the specified datum. In this case the composition of the small displacement torsor of the substitute feature and of the datum are sufficient to give the tolerance zone position.

The positioning of a tolerance zone relative to a datum frame is equivalent to the aggregate some parallel assembly links. In a similar way than the one used in assembly, the algorithm which permits to calculate the position of the part is based on the undeterminated components of each gap (their degree of freedom) in function of the complementary gap components of the other links [Ballot, 1995].

In the case of a datum frame, standard indicate explicitly the hierarchy of the contacts. Then, the specification shown in the figure 5 is treated with the following method: positioning relative to the feature 2 (suppression of the mobility in rotation around x and y and in translation along z) then suppression of the supplementary degrees of freedom between the features 3 and 4 (by example ***J[tx,3,13] = J[ty,3,13] = J[tx,4,14] = 0*** or one of the three other possibilities given by the compatibility system.

-J[ty,3,13] + J[ty,4,14] + J[tx,4,14] - J[tx,3,13] *- dr[3,B].Sin[t[3,13]] + dr[4,B].Sin[t[4,14]] + w[3,B] - w[4,B] + (b[2,B] - b[3,B]).z[3,13] + (-b[2,B] + b[4,B]).z[4,14] + (61.(-(Cos[t[3,13]].dr[3,B]) + Cos[t[4,14]].dr[4,B] + v[3,B] - v[4,B] + (-c[2,B] + c[3,B]).z[3,13] + (c[2,B] - c[4,B]).z[4,14]))/20 = 0*

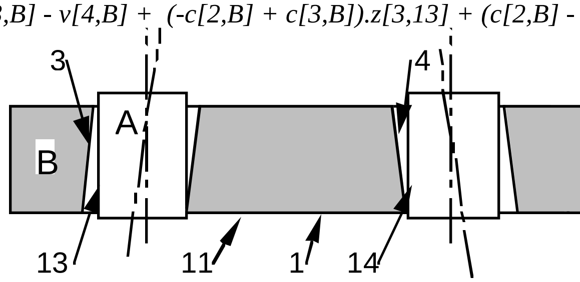
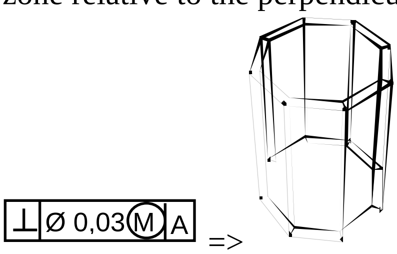


Figure 5 *: reference frame of a localization*

3.5 Tolerance zone : expression of a functional requirement

A geometric specification defines, by the mean of its symbols, the feature on which the specification is applied, and the orientation, the form and the size of the tolerance zone. We will use this information to calculate the deviation area which models the specification. The tolerance zone is determined, on each point of the zone, with the deviation calculated between the substitute feature and the nominal feature; Figure 6, created with the software [Wolfram, 1991] represents the set of deviation components which represent the tolerance zone relative to the perpendicularity.



=> => equation

$$z\alpha - \delta r \le 0.015,\quad -z\alpha - \delta r \le 0.015$$

$$z\beta - \delta r \le 0.015,\quad -z\beta - \delta r \le 0.015$$

$$z\left(\frac{\alpha}{\sqrt{2}} - \frac{\beta}{\sqrt{2}}\right) - \delta r \le 0;0015,\quad z\left(-\frac{\alpha}{\sqrt{2}} - \frac{\beta}{\sqrt{2}}\right) - \delta r \le 0;0015$$

$$z\left(\frac{\alpha}{\sqrt{2}} + \frac{\beta}{\sqrt{2}}\right) - \delta r \le 0;0015,\quad z\left(\frac{\alpha}{\sqrt{2}} + \frac{\beta}{\sqrt{2}}\right) - \delta r \le 0;0015$$

Figure 6 *: model of a cylindrical tolerance zone (increment of π/4)*

4. DEMARCH APPLICATION ON THE EXAMPLE

In order to illustrate our approach and the type of our results, we have applied at the part of the figure 2 the algorithms used for the behavior law study.

4.1 Part model and notations

We are working here on the numerated surfaces shown on figure 7. At each surface is associated a deviation torsor, at each frame feature is associated a gap torsor and a part torsor is associated to the actual part to express the displacements of the part relative to the datum frame.

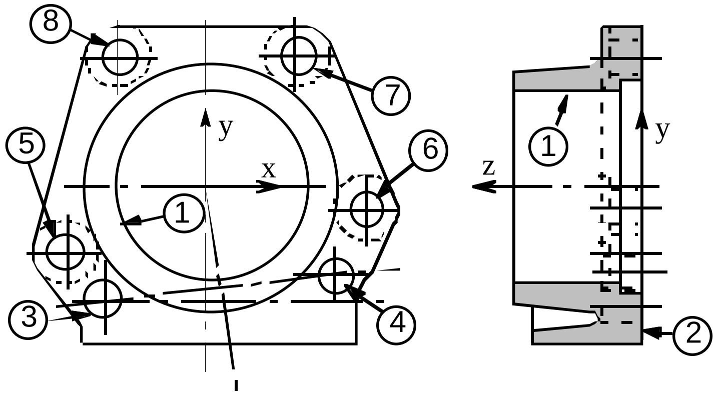


Figure 7 *: substitute feature locations*

4.2 Result examples

Specification	*Generic inequality representative of the deviation*
Ø58H8 Ⓔ	∂r[1, B] ≤ 0.046 ∂r[1, B]≥ 0
⊥ | Ø0,03 Ⓜ | A	(z0-z1).(cost (c[1,B]- c[2,B) - sint (b[1,B] - b[2,B])) - ∂r[1,B] ≤ 0,015
⌖ | Ø0,05 Ⓜ | A | B Ⓜ	(-122.z.c[2,B].Cos[t] + 122.z.c[5,B].Cos[t] + 76.Cos[t-t3].dr[3,B] - 76.Cos[t+t3].dr[3,B] + 46.Cos[t-t4].dr[4,B] + 76.Cos[t+t4].dr[4,B] - 122.dr[5,B] + 122.z.b[2,B].Sin[t] - 122.z.b[5,B].Sin[t] + dr[3,B].Sin[t-t3] - dr[3,B].Sin[t+t3] - dr[4,B].Sin[t-t4] + dr[4, B].Sin[t+t4] - 122.Cos[t].v[4,B] + 122.Cos[t].v[5,B] + 2.Cos[t].w[3,B] - 152.Sin[t].w[3,B] - 2.Cos[t].w[4,B] + 30.Sin[t].w[4,B] + 122.Sin[t].w[5,B] + 2.b[2,B].Cos[t].z[3,13] - 2.b[3,B].Cos[t].z[3,13] - 152.b[2,B].Sin[t].z[3,13] + 152.b[3,B].Sin[t].z[3,13] - 2.b[2,B].Cos[t].z[4, 14] + 2.b[4,B].Cos[t].z[4,14] + 122.c[2,B].Cos[t].z[4,14] - 122.c[4,B].Cos[t].z[4,14] + 30.b[2,B].Sin[t].z[4,14] - 30.b[4,B].Sin[t].z[4,14])/122 ≤ 0,015

Table III *: Geometrical behavior representative of the behavior laws examples*

Specification	*Deviation area*
⊥ | Ø0,03 Ⓜ | A	$30(\gamma(1,B)-\gamma(2,B))-\delta r\le 0.015,\quad 30\left(-\frac{\beta(1,B)-\beta(2,B)}{\sqrt{2}}+\frac{\gamma(1,B)-\gamma(2,B)}{\sqrt{2}}\right)-\delta r\le 0.015$
	$30(-\gamma(1,B)+\gamma(2,B))-\delta r\le 0.015,\quad 30\left(-\frac{\beta(1,B)-\beta(2,B)}{\sqrt{2}}-\frac{\gamma(1,B)-\gamma(2,B)}{\sqrt{2}}\right)-\delta r\le 0.015$
	$30(-\beta(1,B)+\beta(2,B))-\delta r\le 0.015,\quad 30\left(\frac{\beta(1,B)-\beta(2,B)}{\sqrt{2}}-\frac{\gamma(1,B)-\gamma(2,B)}{\sqrt{2}}\right)-\delta r\le 0.015$
	$30(\beta(1,B)-\beta(2,B))-\delta r\le 0.015,\quad 30\left(\frac{\beta(1,B)-\beta(2,B)}{\sqrt{2}}+\frac{\gamma(1,B)-\gamma(2,B)}{\sqrt{2}}\right)-\delta r\le 0.015$

Table IV *: Example of representative area of the specifications*

5. ANALYSYS OF THE SPECIFICATIONS

The definition of the deviation zone permits the integration of the already defined specifications to the chain of dimension. The calculation of the chains of dimension in which some specifications are already included constitutes a part of the specification validation: the functional specification respect.
However, it may be of use to examine the coherence of the specifications on a part independently of the mechanism in which the part is included. The proposed model permits to realize three kinds of specification analysis help.

5.1 Non consistence of a datum frame relative to a tolerance zone

Every specification for which the datum frame does not define the location of the tolerance zone has to be considered like incorrect. If an uncontrolled mobility leaves on the location of the tolerance zone, this one will be ambiguous.
In the proposed model, the undetermined components of the deviation and gap torsor which may stay in the deviation area show this lack of consistence. Indeed, the presence of one of this type of component shows that the deviation area may be deform without any control.

5.2 Extremum search of deviation component

The proposed model represents the geometric specifications by the way of a set of a non linear equations or of a set of linear constraint combinatory (in function of the contact configurations between the substitute element and the datum). Within this context and using a optimization algorithm, it is possible to search the extremum values of the select deviation components.

These values permit to check that all deviations are bounded (the set of geometric specification is suffisant), that the constraints relative to each specification are satisfied by at least one deviation value (the complete set of specification is necessary).

6. CONCLUSION

We have shown that the deviation torsor, the gap torsor and the part torsor associated to the undetermined components of the small displacements provide a coherent model of the geometric specification representation This systematic model gives the representation of the geometric specification as explicit mathematical.

This mathematical expression permits to tackle the questions relative to the specifications consistence with a point of view which ought to be developed. Indeed, the methods proposed to help the validation of the specifications on a part constitute some research axis which still need to be studied closely and completed.

7. REFERENCES

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