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A Computation Method for the Consequences of Geometric Errors in Mechanisms

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ABSTRACT: The work presented here goes along the research for the principles that, starting from functional requirements, allow to compute the nature and value of tolerances on each part of a mechanism. In comparison with A.Clément's or J.Turner's works, our contribution is included in the formal description of the elements of tridimensional tolerance chains. This approach is built upon two elements, a modelization of geometric errors and a method of computation for their propagation inside of a mechanism. The modelization of geometric variations proposed here is founded upon the association of small displacement torsors to the different types of deviations that can be met in a mechanism. From then on, determining the parts' small displacements under the effect of deviations and of gaps of the parts in a mechanism, becomes a computation of the composition of the modelized geometric errors. This computation of each part's position yields two results. First, the formal determination of the part's position in the mechanism in relation with the chains of influent geometric variations influenced by the parts' surfaces. Then, the description of a combinatory of a mechanism's configurations. The application of this method shows the results obtained as well as the possibilities of extension towards a tolerancing aiding tool.

KEYWORDS: modeling of geometric errors, tolerance determination, tolerance modeling, mechanism theory.

1. A NEED : TO ESTABLISH A CONNECTION BETWEEN FUNCTIONAL REQUIREMENTS AND PARTS' GEOMETRIC ERRORS

Geometric tolerances of the parts in a mechanism reduce the deviations so that they correspond to the functional requirements : mounting, gap, contacts.... These prerequisites, expressed under the form of geometric deviations, take on different values according to each part's variations. The impact of these unavoidable geometric errors on the functional requirements then have to be assessed so as to check the congruity of a mechanism to its definition. That is why the relationship between a functional requirement and the different geometric errors and gaps in a mechanism have to be searched for, so as to be able to deduce the indispensable geometric tolerances for each part. It is not always so. It is actually frequently more convenient to start from a tolerance solution and then check that it guarantees the functional requirements.

1.1. The classic approach : assessment of geometric tolerances

This first approach yields two computation methods: the analysis of tolerances and the synthesis of tolerances. The assessment of tolerance is therefore carried out as follows :

- A geometric tolerance solution is proposed by an expert or a designer. It may or may not contain tolerance values.
- A model of the mechanism is established from these geometric tolerances : constraints due to the tolerance, constraints connected to contacts and to the non-interpenetration of parts.
- Two methods can be used. One checks that the tolerance values entail gap values that are compatible with the functional requirements. It is the analysis of tolerances. One can also calculate an optimal technico-economical parting of tolerance values, that will then define the functional requirements. It is the synthesis of tolerances.

This approach is the most current and it can be found in numerous works that rely on a one-directional or a tridimensional modelization of the geometry [6] or [7]. The main interest but also drawback of this approach is in the predetermination of the structure of geometric tolerances. If this predetermination allows to free oneself of the problem of the computation of the function that connects the geometric errors to the functional requirement by replacing it by a function that results from the tolerances, it also yields a restriction of the initial problem. The algorithm of optimization, analysis or synthesis of tolerances then implicitely determines the connections between the geometric tolerances and the functional requirements.

1.2. A complementary approach: determining geometric tolerances

This second approach, less developed today because more recent, precedes the assessment of the tolerance in a complementary manner. Instead of hypothezising the existence of geometric tolerances, they are now going to be calculated on the basis of the functional requirements and the structure of the mechanism. One one-directional example of this can already be found in Pierre Bourdet's method of Δl [5]. The determination of the tolerance is then carried out as follows :

- A modelization of geometric errors is associated to the mechanism's parts.
- The working requirements are gathered. A mechanism can then be studied for all its configurations or for some particular working configurations.
- The laws of geometric propagation of variations inside of the mechanism are then computed: they determine the relationships between functional requirements and geometric variations.
- The mathematic expression of the tolerances is carried out by the separation of the geometric behavior laws on each part.
- These geometric tolerances are assessed classically to compute tolerance values.

This building of tolerances from the influent geometric variations on functional requirements yields a sufficient conditions for minimal tolerance chains. We are now going to develop on this type of approach and more specifically on the way to compute the link between geometric deviations and functional requirements inside of a mechanism.

2. A MODELIZATION OF GEOMETRIC ERRORS : SMALL DISPLACEMENT TORSORS

The modelization of geometric errors adopted relies on several hypotheses that have to be clarified before putting it into practise. These hypotheses were presented in details at the seminar before [4].

2.1. A modelization of geometric deviations

A geometric deviation is measured between at least two surfaces or more if a datum reference frame exists. Nevertheless, from a computation point of view, it seems more interesting to use a model where a geometric deviation is associated to each surface, thus avoiding to describe the combinatory of deviations between all the surfaces of a part. The deviation then represents the difference between the part's nominal surface and a surface of the same nature, tangent and external to the real surface. This surface is named substitution surface. The geometric variations and tolerances that will be computed hence correspond to a combinaiton of deviations.

Another specificity of the model is to differentiate the componants of the geometric deviations of the surfaces. Actually, the surfaces used for the mechanical connections most of the time have properties of invariance in relation with certain translations or rotations. The invariance properties of these surfaces are *a fortiori* verified for small deviations. We will then distinguish the deviations that leave the surfaces invariant from others as they play a particular role in the problem of positioning by contact. We will call these variables undetermined.

Furthermore, because of the low amplitude of the part' displacements caused by the geometric deviations, we use a linearization of these by small displacement torsors. This goes along many other works among which A.Clément's [6].

2.2. The three categories of torsors in the model

The modelization of a mechanism and of the parts that it is made of, relies on three categories of deviation torsors.

The first is that of the geometric deviations between substitution and nominal surfaces. These deviations are represented by a torsor whose shape is defined by the nature of the surface. For instance, the deviation torsor at a point o of a plane s of a part P and of normal $\vec{z}$ is described by the torsor (1). The variables symbolized by i are the componants of the small displacements that leave the surface globally invariant. The shape of the varied deviation torsors is in thesis [1]. The influence of association of these deviations on functional requirements is researched so as to define their limits and then tolerance them.

$$\{T_{s/P}\} = \begin{Bmatrix} \alpha(s,P) & i_{tx}(s,P) \\ \beta(s,P) & i_{ty}(s,P) \\ i_{rz}(s,P) & w(s,P) \end{Bmatrix}_o \quad (1)$$

The second category corresponds to the deviations between parts, i.e gaps. Their shape is determined by the mobility and positioning degrees authorized by the contact. For instance, a contact that follows a line that passes by o between a plane of normal $\vec{z}$ and a cylinder of axis carried by $\vec{y}$; leads to the gap torsor (2) where J is a gap componant and Ind an undetermined componant. The form of gap torsor is computed by different methods [1].

$$\{T_{1/2}\} = \begin{Bmatrix} Ind_{rx}(1,2) & Ind_{tx}(1,2) \\ j_{ry}(1,2) & Ind_{ty}(1,2) \\ Ind_{rz}(1,2) & j_{tz}(1,2) \end{Bmatrix}_o \quad (2)$$

Lastly, each part is submitted to a small displacement because of its positioning by its substitution surfaces, by those of other parts, by the gaps and the position of the other parts. A torsor is then associated to each part, named part torsor, whose componants are to be computed according to the gaps and deviations of the mechanism and by the two following principles.

3. TWO COMPUTATION PRINCIPLES FOR THE PROPAGATIONS OF DEVIATIONS IN MECHANISMS : COMPOSITION AND AGREGATION

The small displacement torsors give us a model of the deviations that distinguish a mechanism made of nominal parts from one that has parts with deviations. We are now going to use this representation to compute the consequences of these deviations on the positioning of the parts first through elementary positioning chains and then globally. After expressing these positions with regards to the deviations and gaps of each part, the relation between a functional requirement and part's deviations can be established.

3.1. Composition of an elementary chain of deviations

An elementary chain of deviations is the succession of small deplacements that occur in a link between two parts in contact with each other by a single gap torsor. An elementary chain of deviations is then composed around each contact modelized by a gap torsor. By breaking up this gap torsor between the surfaces that make up the contact, i.e the surface i of the part $P1$ and j of the part $P2$, the small displacement relation is formed (3). The small displacements of the parts are expressed in a common reference noted R associated to the nominal.

$$\{T_{i/j}\} = \left(\{T_{i/P1}\} + \{T_{P1/R}\}\right) - \left(\{T_{j/P2}\} + \{T_{P2/R}\}\right) \quad (3)$$

By using relation (3), the expression of the small displacement for part $P2$ in relation with the parameters of the chain of deviations i/j with a k index can then be determined, i.e the deviation between the surface j of the part $P2$, the position $P1$ and the deviation of the surface i of $P1$ as well as the gap between surfaces i and j.

$$\{T^{(k)}{}_{P2/R}\} = -\{T_{j/P2}\} + \left(\{T_{i/P1}\} + \{T_{P1/R}\}\right) - \{T_{i/j}\} \quad (4)$$

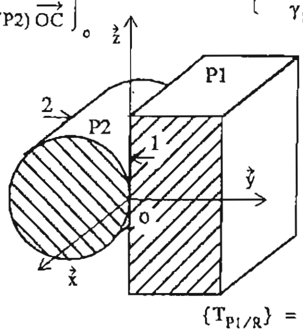
To illustrate the use of this relation, we are going to consider the chain of deviations represented by figure 1.

This elementary chain of deviations shows a contact between parts *P1* and *P2* by a cylindrical surface leaning onto a plane surface.

$$\{T_{2/P2}\} = \begin{Bmatrix} i_{rx}(2/P2) & i_{tx}(2/P2) + \gamma_{2/P2}\overrightarrow{OC} \\ \beta_{2/P2} & v_{2/P2} + dr \cdot \cos\theta \\ \gamma_{2/P2} & w_{2/P2} + dr\sin\theta - i_{rx}(2/P2)\overrightarrow{OC} \end{Bmatrix}_O \qquad \{T_{1/P1}\} = \begin{Bmatrix} \alpha_{1/P1} & i_{tx}(1/P1) \\ i_{ry}(1/P1) & v_{1/P1} \\ \gamma_{1/P1} & i_{tz}(1/P1) \end{Bmatrix}_O$$

$$\{T_{1/2}\} = \begin{Bmatrix} Ind_{rx}(1,2) & Ind_{tx}(1,2) \\ Ind_{ry}(1,2) & J_{ty}(1,2) \\ J_{rz}(1,2) & Ind_{tz}(1,2) \end{Bmatrix}_O$$

$$\{T_{P2/R}\} = \begin{Bmatrix} \alpha_{P2/R} & u_{P2/R} \\ \beta_{P2/R} & v_{P2/R} \\ \gamma_{P2/R} & w_{P2/R} \end{Bmatrix}_O$$



$$\{T_{P1/R}\} = \begin{Bmatrix} \alpha_{P1/R} & u_{P1/R} \\ \beta_{P1/R} & v_{P1/R} \\ \gamma_{P1/R} & w_{P1/R} \end{Bmatrix}_O$$

Figure 1 : example of an elementary chain of deviations between a cylinder 2 and a plane 1

The application of relation (4) allows to determine[1] the expression (5) of the torsor of part *P2* according to the characteristics of the contact, the deviations and the small displacement of part *P1*. Two componants of *P2*'s small displacement then appear as determined. Actually, the componants that are not circled contain undetermined values and are thus degrees of mobility of *P2/P1*.

$$\{T^{(1)}{}_{P2/R}\} = \begin{Bmatrix} \alpha_{1/P1} + \alpha_{P1/R} - i_{rx}(2/P2) - Ind_{rx}(1,2) & i_{tx}(1/P1) + u_{P1/R} - Ind_{tx}(1,2) + \ldots \\ i_{ry}(1/P1) + \beta_{P1/R} - \beta_{2/P2} - Ind_{ry}(1,2) & v_{1/P1} + v_{P1/R} - (v_{2/P2} + dr) - J_{ty}(1,2) \\ \gamma_{1/P1} + \gamma_{P1/R} - \gamma_{2/P2} - J_{rz}(1,2) & w_{P1/R} - w_{2/P2} - Ind_{tz}(1,2) + \ldots \end{Bmatrix}_O \quad (5)$$

We have expressed the position of a part in a mechanism that has geometric deviations. Nevertheless, a part is not always positioned only by a link.

3.2. Agregation of elementary chains of deviations

In the general case, there exist q links between a part P and the surrounding parts. This entails q possible expressions of the small displacement torsor, according to the varied gap, position and deviation parameters of each elementary chain of deviations. Every one of these q links yields the suppression of a certain number of degrees of mobility and hence partially positions the part. Similarly, these links contribute to the positioning of the part but also create local mobilities, the undetermined. These mobilities will, in the case where

1. For presentation reasons, this expression is simplified by the fact that the surfaces are oriented according to the axes of the coordinate system and an expression point that is the same for all torsors. The method is nevertheless general.

they do not correspond to a mobility of the part, be determined by other elementary chains of deviations.

Determining the global displacement of a part $\{T^*_{P/R}\}$ then consists in posing the equality of the different expressions coming from all the elementary chains of deviations and solving the system (6) in relation with the contact variables undetermined. That is to say, posing the equality of the small displacement expressions and trying to express the local degrees of mobility according to the set of the connection elements.

$$\{T^*_{P/R}\} \equiv \{T^{(1)}{}_{P/R}\} = \{T^{(2)}{}_{P/R}\} = \ldots = \{T^{(q)}{}_{P/R}\} \qquad (6)$$

These $q\text{-}1$ equalities build up a system of linear equations of $m = 6(q\text{-}1)$ lines (6 for each torsor) with n unknowns (the undetermined). Such a system is generally potentially over or under determined. This means that there exist undetermined variables whose value cannot be calculated (cinematic mobility degrees) and others for which there will on the contrary be an over-abundance of definitions (hyperstatism degrees). In such a context, the only possible solution is given by the Gauss method of the partial pivot. Its application then leads to a system whose shape is given here-under.

$$\left.\begin{array}{r} \left.\begin{array}{r} a_{11}Ind_1 + a_{12}Ind_2 + \ldots\ldots + a_{1n}Ind_n = b_1 \\ c_{22}Ind_2 + \ldots\ldots + c_{2n}Ind_n = \bar{b}_2 \\ \vdots \\ k_{rr}Ind_r + \ldots + k_{rn}Ind_n = \bar{b}_r \end{array}\right] (1) \\ \left.\begin{array}{r} 0 = \bar{b}_{r+1} \\ \vdots \\ 0 = \bar{b}_m \end{array}\right] (2) \end{array}\right. \qquad (7)$$

The first part of system (7), of rank r, allows to calculate $\{T^*_{P/R}\}$ by substituting the r undetermined expressions in one of the expressions of the small displacement of the part $\{T^{(k)}{}_{P/R}\}$.

The second part of the system gives a set of compatibility conditions that, if they are verified, validate the expression of the small displacement. The solution for a system of undetermined values for each part of a mechanism, then the gathering of the expressions of the small displacement componants of all the parts allow to calculate each part's small displacement by recurence. The parts' small displacements thus determined then communicate the influent deviations and only these. The others are actually eliminated by an algebraic simplification of the expressions. It can also be noted that the orientation of the elementary chains of deviations is not necessary. The problems of anteriority of positioning can hence be forgotten.

3.3. Properties of the compatibility system

The conditions of compatibility in system (7) only contain distorsion and gap componants. With this system, on the one hand the meaning has be looked for by placing the modelization inside of the theory of mechanisms; on the other hand, the constraints it entails on the propagation of deviations in a mechanism have to be determined.

The proposed formulation holds an equivalence between the undetermined variables and the cinematic variables. But the theory of mechanisms shows that the degree of hyperstatism of a mechanism is : $H = |M_s - M_c|$ where M_s the static mobility is[1]: $M_s = -6(q-1) + n = n - m$ and M_c the cinematic mobility is : $M_c = n - r$. Part 2 of system (7) is then found to correspond to the degrees of hyperstatism of the mechanism. Furthermore, the model completes the theory of mechanisms by bringing an explanation of the geometric conditions induced by every degree of hyperstatism thanks to the formal solution of equalities (6).

These conditions have to be verified whatever the value of the parts' geometric deviations, which is defined as random. The links between gaps are hence searched for. A new matricial writing is used (8), equivalent to the compatibility conditions (7) and where *E* is the set of deviations and *J* that of gaps.

$$0 = B(J, E) \Leftrightarrow C \cdot J = D(E) \qquad (8)$$

This system contains more gap componants than equations. It is under-determined. There is therefore a combinatory of the sets of gap componants from which the value of the other gap componants can be calculated. This yields a subset of the gap componants whose value can be imposed (in general 0 which means contact). The other componants are then calculated by the resolution of system (8). Every one of these sets defines a configuration of the mechanism, i.e a typology of the contact between parts (dominant leaning plane,...)

The development of the combinatory of the sets of possible gap solutions ensures the numbering of all the configurations of a mechanism, hence of all the potential behaviors for tolerancing. Those that correspond to the working are then chosen.

But system (8) can also be regarded as a set of constraints to calculate the case of the most unfavorable working, by optimization. This approach is mostly useful if the parts are not solidary and if the set of configurations corresponding to the working is not known.

3.4. An example of the search for the extreme configurations of a mechanism

The result of the calculation of the configurations is illustrated by the example of the rail represented in figure 2. The application of both the hereabove-defined principles, programmed into a formal computation software, allows to obtain the displacements of parts B and C as well as the compatibility system (9).

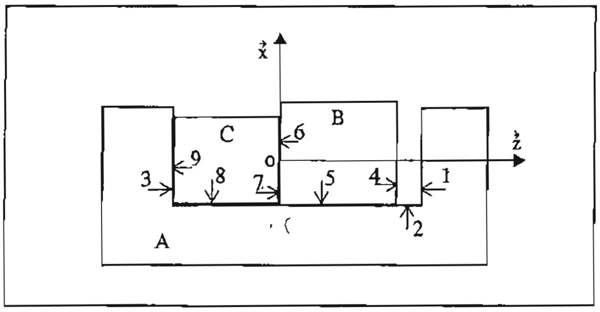


Figure 2 : definitions of the parts and notations of the rail

1. The notations used here are the same as in system (7).

As an example for the reading of a compatibility system: the second equation that follows shows that there exists an over-abundance of orientations around the axis $\vec{y}$, between the links [8,2] and [9,3] of the parts A and C.

$$0 = -\beta[2, A] + \beta[5, B] + \beta[3, A] - \beta[6, B] + \beta[7, C] - \beta[9, C] - J[ry, 5, 2] + J[ry, 6, 7] + J[ry, 9, 3]$$
$$0 = \beta[2, A] - \beta[8, C] - \beta[3, A] + \beta[9, C] + J[ry, 8, 2] - J[ry, 9, 3] \qquad (9)$$

This system of 2 equations with 4 gap unknowns in the orientation is solved by fixing 2 of the 4 gaps in 0, which expresses the orientation identity given by the contact. Each one of the 2 remaining gaps is then calculated according to the orientation deviations of the planes contained in each equation. These configurations describe the set of types of *compatible* contacts. Every contact of a configuration can then be described in terms of partial or full contact as proposed by Shodi and Turner [7].

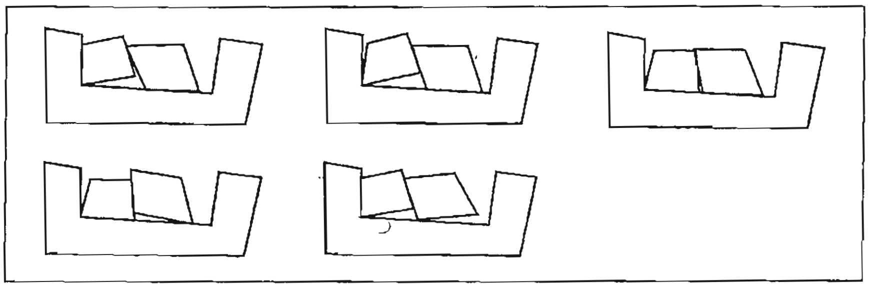


Figure 3 : configurations determined from the compatibility system of the mechanism

The computation of the law of propagation of deviations is carried out after the selection of one or more of the configurations that represent the mechanism's behavior. In the case where working configurations are not known, system (9) will be added as a constraint when computing tolerance values.

4. DETERMINATION OF THE PROPAGATION OF DEVIATIONS INSIDE OF A MECHANISM

With the expression of the parts' small displacements, the propagation of deviations inside of a mechanism can be calculated, which is our initial target. This will also allow us to reach the corresponding geometric tolerances.

4.1. Calculation of a functional requirement as a function of deviations

The knowledge of the parts' displacements in relation with deviations allows to express a functional requirement in two operations: a new composition operation of the small displacements as well as a localization and orientation operation. For a functional requirement *l* between the surface *i* of part *P1* and *j* of part *P2*, the composition of the small displacements is carried out and then, the comoment with the torsor of the pluckerian coordinates $\{P^{(l)}{}_{i/j}\}$. This torsor gives the direction of the deviation and is expressed in all points of the domain where the functional requirement $f_{i/j}(l)$ is exerted.

$$f_{i/j}(l) = \{T^*_{i/j}\} \cdot \{P^{(l)}_{i/j}\} = \left[\left(\{T_{i/P1}\} + \{T^*_{P1/R}\}\right) - \left(\{T_{j/P2}\} + \{T^*_{P2/R}\}\right)\right] \cdot \{P^{(l)}_{i/j}\} \quad (10)$$

As an example, function (11) represents the expression of a functional requirement according to the axis $\vec{z}$ between surfaces 4 and 1 of the mechanism in figure 2. This mounting requirement, $f_{1/4}(l) > 0$ is relative to a configuration of the mechanism and to one point parametered by xc[4, 1] and yc[4, 1].

$$u[1, A] + u[3, A] + u[4, B] + u[6, B] + u[7, C] + u[9, C] + (-\beta[5, B] + \beta[8, C] + \gamma[6, B] - \gamma[7, C])\, x[6, 7] + (\beta[2, A] - \beta[8, C] - \gamma[3, A] + \gamma[9, C])\, x[9, 3] + (-\beta[2, A] + \beta[5, B] + \gamma[1, A] - \gamma[4, B])\, xc[4, 1] + (\beta[1, A] + \beta[3, A] + \beta[4, B] + \beta[6, B] + \beta[7, C] + \beta[9, C])\, yc[4, 1] > 0 \quad (11)$$

The formal expression of a functional requirement is thus obtained in all points of a domain by its relation to the deviations of the parts of a mechanism. *This expression is minimal for, after simplification, it contains only the deviation parameters that directly concern the functional requirement.* In the case of partial contacts, it also contains contact parameters that are useful when computing tolerances.

4.2. Resultant mathematic tolerance in deviation space

We formally know the causes of the positioning of the parts of a mechanism according to the parts' deviations. In this context, determining the tolerances of the parts only means distinguishing the terms of the deviations according to the part they belong to. A set of constraints can thus be determined : they build up a mathematic structure of geometric tolerances. *These mathematic tolerances are constraints that are both necessary and sufficient for they correspond exactly to the section of the functional requirement the part fills.* This results in tolerances with no transfer.

For instance, for part B, constraints (12) represent the tolerance that corresponds to the functional requirement $f_{4/1}(l)$ as well as the resultant of the partial contact between surfaces 6 and 7. Where t and t' are the values of tolerance that will be computed with respect of functional requirements.

$$u[4, B] + u[6, B] + (\gamma[6, B] - \beta[5, B])\, x[6, 7] + (\beta[5, B] - \gamma[4, B])\, xc[4, 1] + (\beta[4, B] + \beta[6, B])\, yc[4, 1] < t$$
$$\&\&$$
$$(-\beta[5, B] + \gamma[6, B])\, (-x[6, 7] + xc[6, 7]) >= t' \quad (12)$$

These constraints apply in all points of their respective surfaces; the first constraint therefore has to be distributed upon surface 4 and the second one upon surface 6. Because of the linearization of deviations, only the constraints at the edges has to be expressed. In the next paragraph, the modelization of a standardized geometric tolerance will allow us to show the result of this computation via an example.

4.3. The modelization of standardized geometric tolerance

Even if the method proposes a solution for a direct geometric tolerance, it can be of interest to use the principles developed here to analyse or synthesize another tolerancing solution, while still keeping the function of propagation of deviations. The two composition principles, and potentially that of agregation, of the elementary chains of deviations are applied to the datum surfaces of the substitution part on the nominal part. In this context, the functional requirement computed is the tolerance zone itself. These imposed tolerances are expressed mathematically and then used instead of the direct tolerances, even if they are more demanding.

Figure 4 gives an example of the modelization resulting from a dimensional tolerance. The 8 constraints are the result of the application of 2 tolerances (minimum and maximum dimension) on the 4 corners of the rectangular ends of the part.

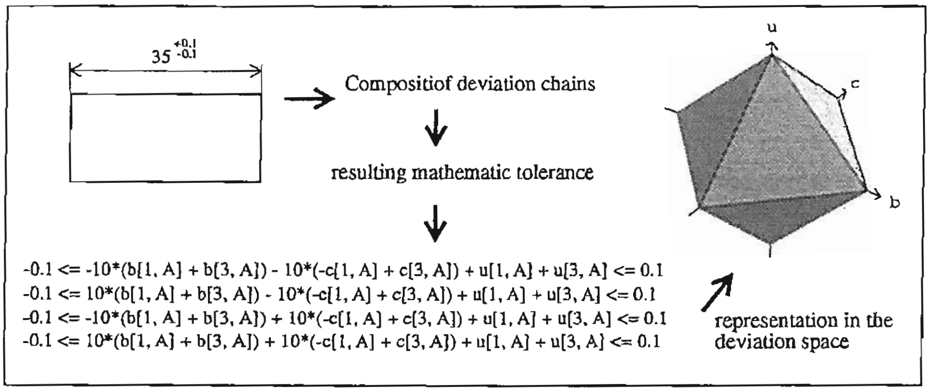


Figure 4 : modelization of a dimensional tolerance

5. CONCLUSIONS AND PERSPECTIVES

The method developed here allows to systematically determine the influence of geometric errors on the small displacement of parts in a mechanism. It therefore uses two original principles of composition and agregation of the elementary chains of deviations. These principles are founded upon the introduction of undetermined variables. The knowledge of the causes of the small displacements of a mechanism's element, in terms of geometric deviations, not only allows to formally express the functional requirements but also opens onto a mathematic tolerance. This tolerance, expressed in the deviation space, is both necessary and sufficient as regards the functional requirement. The results obtained for the modelization of geometric tolerances also seem to compare with those obtained by A.Ballu and L. Mathieu [3] in the field of control.

Apart from the examples mentioned here, the proposed methodology has already been positively put into practise in numerous complex cases, including industrial ones[1]. Trials have also been carried out in the field of manufacturing tolerancing and in that of the modelization of positioning in manufacturing [2].

To this aim, the concepts have been programmed with the help of a formal computation software in a computerized model [9]. The current orientation of our work hence concerns the possibilities of extension towards a tolerancing aiding tool by adjoining specific methods of analysis and synthesis of tolerances. The second subject of research is the translation of these mathematic geometric tolerances into their standardized expression. Actually, such a translation will not always be possible, which will obviously lead to a questioning of the possibilities of expressions of standards.

6. REFERENCES

(1) Ballot, E., 1995, Lois de comportement géométrique des mécanismes pour le tolérancement, thèse de doctorat de l'Ecole Normale Supérieure de Cachan

(2) Ballot, E., Bourdet P., 1996, Chaînes de cotes 3D : présentation par l'exemple, Science et Technologies, v 68, 23-27

(3) Ballu, A., Mathieu, L., 1995, Univocal expression of functional and geometrical tolerances for design, manufacturing and inspection, 4th CIRP International Seminar on Computer Aided Tolerancing, Tokyo, Japan

(4) Bourdet P., Ballot, E., 1995, Geometrical behaviour laws for computer-aided tolerancing, 4th CIRP International Seminar on Computer Aided Tolerancing, Tokyo, Japan

(5) Bourdet, P., Remy-Vincent, J., 1991, Schneider, F., Tolerance analysis in manufacturing, CIRP International Working Seminar on Tolerancing, Penn State

(6) Clément, A., Desrochers A., Rivière, A., 1991, Theory and practice of 3-D tolerancing for assembly, CIRP International Working Seminar on Tolerancing, Penn State

(7) Sodhi, R., Turner, J., 1994, Relative positionning of variational part models for design analysis, Computer-aided design, vol. 26, N° 5, 366-378

(8) Turner, J. U., 1993, A feasibility space approach for automated tolerancing, Journal of Engineering for Industry, vol. 115, 341-346

(9) Wolfram, S., 1996, Mathematica : A system for doing mathematics by computer, Addison-Wesley, Redwood City