COMPUTER-AIDED TOLERANCING

EDITED BY FUMIHIKO KIMURA

Theory and practice of tolerances are very important for designing and manufacturing engineering artefacts on a rational basis. The introduction of computer-aided methods has led to new developments, which are presented in this book.

Computer-aided Tolerancing comprises a collection of contributions covering key technologies including:

functional tolerance description
formal modeling of tolerancing
modeling of geometrical errors and variations
statistical tolerancing
computer support systems for tolerancing
computational metrology and its relationship with tolerancing.

This book provides an excellent introduction for anyone interested in computer-aided tolerancing, as well as CAD/CAM users who wish to use the tolerancing capability in CAD/CAM systems. It will also serve as a good starting point for advanced research activity. The book is derived from the seminar on Computer-aided Tolerancing, organized by CIRP, JSPE and ASME in Tokyo, Japan 1995.

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Geometrical Behavior Laws for Computer-aided Tolerancing

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Abstract

The analysis of the difficulties encountered with traditional dimension chains in the description of the behaviour of a set of parts, with variation, has led us to develop a tridimensional model of variations. It is therefore designed to treat the problem of transfer of dimensions.

The definition of the model of the variation on surfaces of parts relies on a 2-type classification of defaults which allows to then connect them formally.With this model, the requirements of tolerancing have brought about a definition of two operators. The first determines the specifications that may be found on a part by defining the intersection of the domains of deviation of surfaces in relation with their nominal model. The second calculates the union of motions of a part under the influence of each of the mating faces with contiguous parts.

The implementation of the model and of the two operators then allows for a definition of the varied possible configurations of the mechanism. For each of these configurations, the constraints of tolerancing that are exerted on the geometry of each part in a mechanism can be expressed mathematically.

Keywords

Mechanism, tridimensional tolerance chains, geometrical model, dimensioning, mobility degrees.

1 INTRODUCTION.

Though it is theoratically possible to obtain a perfect geometry, one has to admit that for an optimal cost production tools make an approximate geometry. To take the real geometry of a part into account, surface geometric variations have to be limited.

Tolerancing is the means that help designers express these variations.

Among the varied existing theories on tolerancing of parts, some have emphasized mathematical specifications, e.g. A Raquicha's model of shifting (Raquicha, 1983). Others have highlighted the transfer of functional conditions, such as A.Clément's method of technologically and topologically associated surfaces (Clement, 1991) or M.Giordano's and D.Duret's model of gap spaces (Giordano, 1993). Though the varied aspects of tolerancing are all connected, we will concentrate here more on the choice of tolerances.

2 TOLERANCING: NATURE OF THE PROBLEM

The tolerancing of parts of a mechanism must ensure the assembly of the mechanism and the interchangeability of parts.Tolerancing thus has to satisfy two constraints: to respect the functional requirements that set limits for the relative positions between parts, and to take the variations induced by the manufacturing into account.

Our analysis of the way geometric specifications are obtained will therefore start by the analysis of the transfer of functional conditions on the parts of a mechanism.

This transfer is usually calculated with a one-directional model called calculation of tolerance chain.

So as to explain the limits of this one-directional model, we will consider an example of a mechanism. This study will allow to establish the characteristics necessary for a tri-dimensional model of variations that will then be introduced.

2.1 Introductory example

The mechanism represented in figure 1 has several links plane/plane, shaft/hole and two tolerance chains. The orthogonal morphology of the part lets believe that it can be processed with a one-directional tolerance chain. Actually, there is no a-priori coupling between variations belonging to orthogonal directions. An independant processing of tolerance chains along both directions then determines dimensional specifications of each part. In this model, the geometry of the real part is compared with a set of double points or with parallel planes.

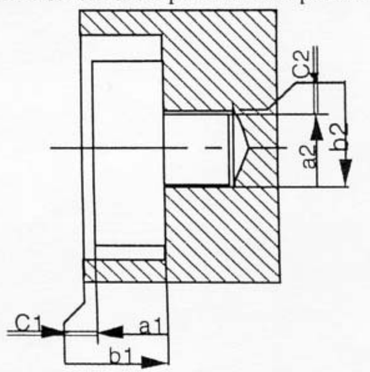

Figure 1 Writing of tolerance chains relative to functional conditions

Figure 2 represents the same mechanism with variations on each part, voluntarily amplified for the sake of demonstration. The real surfaces are modelized by surfaces of the same nature, tangent to nominal surfaces on the free side of the material. Compared with the ideal geometric model, each surface hence shows position, orientation and individual variations (radius variation of the shafts and holes of the example).

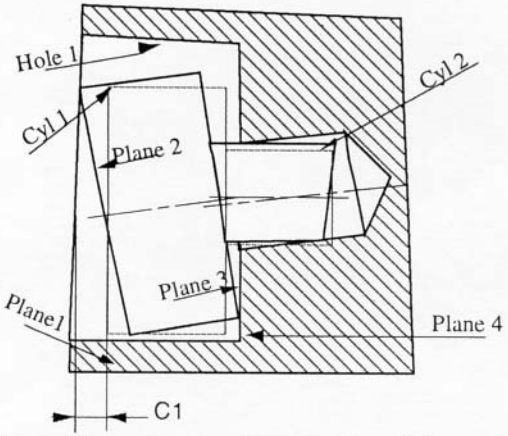

Figure 2 Representation of the mechanism with geometric variations

2.2 Limitations of one-directional models

The one-directional tolerance chain does not take orientation variations of surfaces into account; the proposed example, despite its simplicity, allows to see two defects of one-directional models.

Connection between variations from different directions.

The study of gap, direction after direction, and the orthogonal morphology of the part result in the independance of the specifications of both directions.

In figure 2, it is on the contrary shown that there is an interdependance between the value of gap C1 and the orientation of plane 3 in relation with plane 4; this orientation does not only depend on the flat mating but also on the configuration of the shaft/hole mating.

This figure 2 also illustrates another phenomenon: the influence of the relative position of both parts in the assembly. Actually, if one considers the shaft after a half turn rotation, gap changes value. In the case of turning parts, this implies the research of the most unfavorable values and in that of a fixed jig, the possibility to look for a position that would minimize the influence of variations. One-directional methods never take this type of relationship into account.

The example also shows that there is practically no mechanism that can be processed as a purely one-directional problem.

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Translation of results into normalized specifications.
The results of one-directional tolerance chains can be rather easily translated into dimensional specifications or into simple position specifications. This type of specification nevertheless only represents a part of the potential geometric specifications. The specifications of orientation or localisation, through a reference system cannot be obtained with such a model.

2.3 Parts motion under mating variation.

The calculation of the value for a functional condition between two surfaces of different parts in a mechanism requires the calculation of the position of these surfaces. This requires a determination of the position of each part in relation with the variations of the surfaces in contact with the other parts of the mechanism. Geometric variations are thus propagated from part to part throughout the mechanism.

The function of calculation of the consequence of variations or assembly with variations is essential in the problem of tolerance chains, as have stressed F.Schneider and J.Rémy-Vincent (Schneider, 1993). Numerous works have actually been dedicated to these questions and have offered different approaches:

- Case study is the solution chosen by the S.A.T.T method of A.Clément (Clement, 1991). The function of the calculation lies upon the research for Minimum Reference Geometric Elements.
- Applying rules to compute the suppression of degrees of mobility (Turner, 1992).
- K.Takahashi has imagined an algorithm to assemble parts with variations (Takahashi, 1993). The field of application for this algorithm is restricted to flat surfaces which makes it perfectly suited for the study of polyhedral parts.
- An algorithm of optimization by linear programming has been suggested by J.Turner (Turner, 1990) and completed by R. Sodhi (Sodhi, 1994). These calculations procedures of behavior have to be applied to each case with variations, which leads to a calculation of tolerances by simulation.

Nevertheless, there is no systematic formal method that deals with this problem. We are then going to rely on the existing works and our analysis of the example to propose a formal model of variations for geometric tolerancing.

3 MODELIZATION OF VARIATIONS ON SURFACES OF PARTS

The model developed is founded upon varied hypotheses:

- First, the conservation of the typology of the nominal surface to modelize the real surface. A real surface nominally flat will then be represented by a plane.
- The "perfect" surface representing the real surface will be tangent and out of the material in relation with the real surface of the part. This hypothesis allows for the assembly of parts.
- The little amplitude of the variations regarding the nominal dimensions of the parts allows to modelize the transformation, which associates the surface of the real model to the nominal surface by a small displacement torsor.
- The motion of the parts under the effect of their variations is also a small amplitude motion and can be modelized by a small displacement torsor.

3.1 Two types of variables for variations

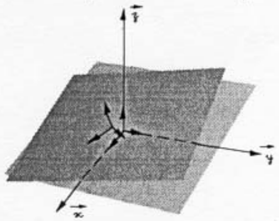
The surfaces traditionally used for mechanical link often have particular invariance properties by translation along an axis or rotation around an axis. A plane is globally invariant for all translations of a colinear vector to the nominal plane and for all rotations on a colinear axis to the nominal plane. This property of invariance for certain usual surfaces such as plane, cylinder, sphere, is all the more easily checked for small displacements.

Let us consider the torsor that modelizes the small displacement that connects a real surface with its nominal surface. It is expressed under its canonic form, in an orthonormed reference that includes the normal of the nominal plane, according to what follows:

$$\{T_{Real/Nominal}\} = \begin{Bmatrix} \alpha & u \\ \beta & v \\ \gamma & w \end{Bmatrix}_O \qquad (1)$$

The components of the small displacement torsor that leave the reference plane globally invariant are called undetermined components *Ind*.

For example, the small displacement torsor of the considered plane is:



$$\{T_{Real/Nominal}\} = \begin{Bmatrix} \alpha & Ind_{tx} \\ \beta & Ind_{ty} \\ Ind_{rz} & w \end{Bmatrix}_O \qquad (2)$$

The components of the transformations of the small displacement that globally leaves the surface invariant are generally arbitrarily fixed, 0 for instance. This 0 value is ambiguous for it hides the notion of invariance in relation with true 0 variations.

As these components of small displacements are always undetermined, in the reference associated with the surface and in relation with the considered surface, it means that they have the following properties:

$$\forall a \in \Re;\ a + Ind = Ind \qquad \forall a,b \in \Re^2;\ a.Ind + b.Ind = Ind \qquad (3)$$

In the model we are considering, these components are considered as undetermined parameters as regards the considered displacement. Hence, they are exogenous parameters as regards a small displacement but not necessarily exogenous as regards the whole part or mechanism. This property will be used by the operators of the model.

3.2 Small displacement torsors of the model

The characterization of the part variations, of the gap between parts as well as of the small displacements of parts, under the effect of variations, and as regards their theoratical relative position, depends on a set of torsors that we are going to define.

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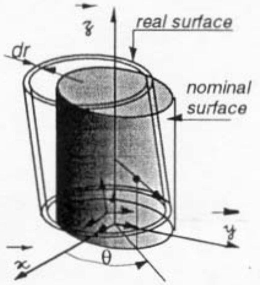
Deviation torsor

A deviation torsor expresses the variation between a nominal surface and a surface of the same nature by modelizing the real surface. This torsor is associated with a surface the same way a $\Delta\ell$ is associated with each surface of a part, (Bourdet, 1973). The deviation torsor is built upon the composition of two torsors: a small displacement torsor for the surface and a torsor of the characteristics variation of the surface. The characteristics variation torsor takes into account the variation of the radius of a sphere or the top angle of a cone, for instance.

In the case of a z direction cylinder, it yields:



$$\{T_{Real/Nominal}\} = \begin{Bmatrix} \alpha & u \\ \beta & v \\ Ind_{rz} & Ind_{tz} \end{Bmatrix}_O + \begin{Bmatrix} 0 & dr\cos(\theta) \\ 0 & dr\sin(\theta) \\ 0 & 0 \end{Bmatrix}_P$$

$$\{T_{Real/Nominal}\} = \begin{Bmatrix} \alpha & u+dr\cos(\theta) \\ \beta & v+dr\sin(\theta) \\ Ind_{rz} & Ind_{tz} \end{Bmatrix}_O \qquad (4)$$

p is a point of the surface parametered by θ

The characteristics variation torsor may be zero when the surface does not have specific characteristics, as is the case for a plane.

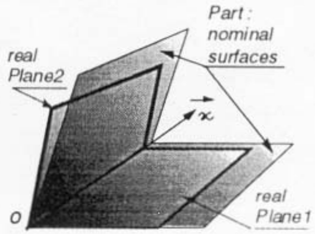
One has to notice that the components of the characteristics variation torsor which are not used, are equal to zero as the taking into account of invariance by motion is done at the level of the small displacement torsor of the surface.

The deviation torsors thus modelize position, orientation and specific variations of each part surface.

Variation torsor

A variation torsor expresses the relative variation between two or more real surfaces of a one part. In the case when it expresses the variation between two surfaces, the variation torsor can be calculated by the composition of the deviation torsors of each surface. This is done with a common reference on both surfaces and with the application of the property (3). To build such a common reference, one can refer to the definitions of EGRM presented in chapter 2 of D. Gaunet's thesis (Gaunet, 1994), or to the operator of intersection of deviation fields presented in paragraph 4.2.

The following formula (5) is an example of a variation torsor between two nominally non parallel planes.



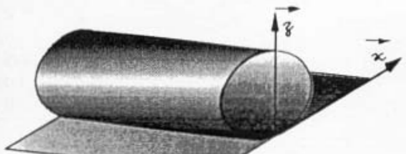
$$\{T_{Plane1/Plane2}\} = \{T_{Plane1/Part}\} - \{T_{Plane1/Part}\}$$

$$\{T_{Plane1/Plane2}\} = \begin{Bmatrix} \alpha_{(1/2)} & Ind_{tx(1/2)} \\ Ind_{ry(1/2)} & Ind_{ty(1/2)} \\ Ind_{rz(1/2)} & Ind_{tz(1/2)} \end{Bmatrix}_O \qquad (5)$$

Gap torsor

A gap torsor expresses the gap between two surfaces of different parts which are nominally in contact. There will be a gap torsor associated to each couple of surfaces of a mechanism that makes a contact. The gap torsor then only concerns two surfaces that belong to different parts. The definition of the gap torsor requires the same operator as before but in a different application.

Actually, the part considered here is an immaterial wedge that formalizes the gap between the two parts. The operator for the research of the intersection fields of deviations is applied to this virtual part only made up of two surfaces. Then, for instance, the gap torsor of an axis x cylinder in contact with a norm plane in z can be determined in the formula (6):



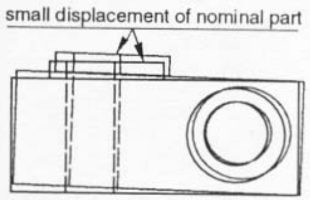
$$\{T_{Plane/Cylinder}\} = \begin{Bmatrix} J_{rx} & Ind_{tx} \\ Ind_{ry} & Ind_{ty} \\ Ind_{rz} & J_{tz} \end{Bmatrix}_O \qquad (6)$$

where *J* represents the components of the gap and *Ind* the undeterrmined components of the mating.

Small displacement torsor per part

The small displacement torsor associated with each part gaps is central in this model. It actually establishes, through all the composition chains of all the small displacements, the connection between all the deviations of a part. It will then be used either to express the displacement of the model of the real part in its assembly with the nominal model, or to calculate the small motion of a part in the mechanism, function of the surface variations of the part in contact with the adjacent surfaces and the small displacements of the neighbouring parts.

Each part will have its associated small displacement torsor in relation with a set of references :



$$\{T_{p/R}\} = \begin{Bmatrix} \alpha_{p/R} & u_{p/R} \\ \beta_{p/R} & v_{p/R} \\ \gamma_{p/R} & w_{p/R} \end{Bmatrix}_O \qquad (7)$$

All these torsors yield the elements necessary to describe the mechanism. We are now going to define the operators that handle these different torsors so as to answer the requirements of geometric tolerancing.

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4 OPERATORS LINKED WITH THE MODEL.

To solve problems of tolerancing by tolerance chains one must be able to answer two questions: what is the set of possible dimensions for each part ? what are the significant positions of parts in the mechanism with variations?

Before showing the answer that we can bring to these two questions, let us develop on the composition of small displacements as shown by an elementary mating.

4.1 Small displacement resulting from an elementary mating

The three torsors of deviation, gap and small displacement per part are connected by a composition relation of small displacements. For the model, an elementary mating is based on the couple of two surfaces in contact with each other. These surfaces may be simple surfaces such as plane or cylinder or sets of surfaces; nevertheless, a single deviation torsor will be associated, by part, with each surface or set of surfaces. Figure 4 illustrates the respective positions of the varied torsors that act in the modelization of a mating.

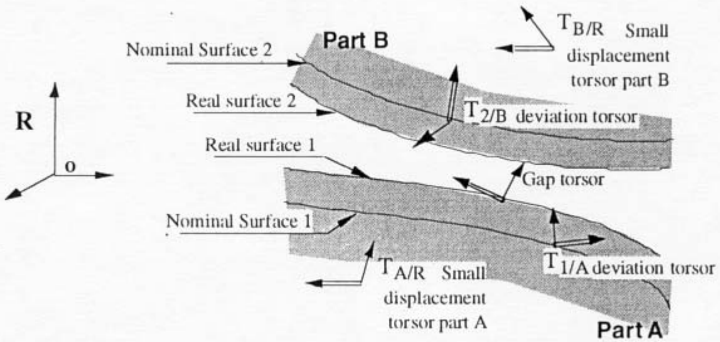

Figure 3 Small displacement torsors of a mating

The composition equations of the small displacement torsors of the elements that build up the mating can be writtenas follows, to express the motion of part B in relation with the mating variations:

deviation torsor real surface 1 — deviation torsor real surface 2

$$T_{B/R} = T_{A/R} + T_{1/A} - T_{2/B} + T_{1/2} \qquad (8)$$

Small deplacement torsor part B — Small deplacement torsor part A — Gap torsor

This relation can be applied similarly for the modelization of either the contact between two parts of a mechanism or that between a real surface and its nominal model.

4.2. Intersection between deviation fields.

The common deviation field between two surfaces or between a set of surfaces is the set of characteristics that can be specified. Formula (8) allows to connect the displacement of the real part as regards its nominal model, and that for each surface.

The characteristics that can thus be specified are the common characteristics of this set of elementary mating. The intersection operator can be built upon the principle of the unicity of the positioning of the part as regards the nominal and for the set of considered matings. The following operator then can be defined:

$$T^{(part)}_{P/R} = \bigcap_{i=1}^{n} T^{(\ell_i)}_{P/R} \qquad (9)$$

where ℓ_i is the displacement due to the mating ℓ_i.

This operator is concretely defined by the unicity of the displacement torsor of the real part as regards its nominal model. The application of the Gauss elimination method to the linear equation system that can be formulated after this principle, shows a set of constraints which are the required conditions for the existence of a global small displacement torsor for the whole part. These conditions are actually the expression of the tridimensional dimension of the part. Variation coefficients then indicate the reference of expression of the variation torsor in its canonic shape.

The tolerancing of a part is equal to the determination of conditions that constraint the model of the real part on the nominal model. This principle, translated by the operator of the intersection field of deviations gives all dimensioning opportunities between n surfaces (n≥2) of a one part. The application of this operator to the gap between two parts calculates the parameters of the gap torsor.

The implementation of this operator has also shown the combinatorics of the possible specifications. There is not one dimensioning solution but a sometimes rather important set of minimal specifications that are simultaneously possible for one part.

4.3 Union of setting chains

The principle on which the model is built is to associate a deviation torsor to each surface of each part. The links between parts are then cut up into couples of mating faces. It is therefore a series of local descriptions, that then have to be related to see the global behavior of the mechanism.

Each elementary mating allows to write the composition of small displacements of the chain relative to the mating. To calculate the displacement of the part relative to all matings, the union of the set of small displacements for each mating has to be realized. An operator that can translate the following definition has to be found:

$$T^{(part)}_{P/R} = \bigcup_{i=1}^{n} T^{(\ell_i)}_{P/R} \qquad (10)$$

where ℓ_i is the displacement due to the mating ℓ_i.

The definition of the union operator uses the property of unicity of the relative motion of the part. It then yields:

$$\forall i,j; T^{(\ell_i)}_{P/R} = T^{(\ell_j)}_{P/R} \qquad (11)$$

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The research for the small displacement of the part due to variations leads us to consider this equality of torsors. After an expression at the same point, as a system of $6(n-1)$ equations, it will be solved in relation with the undetermined values of these matings.

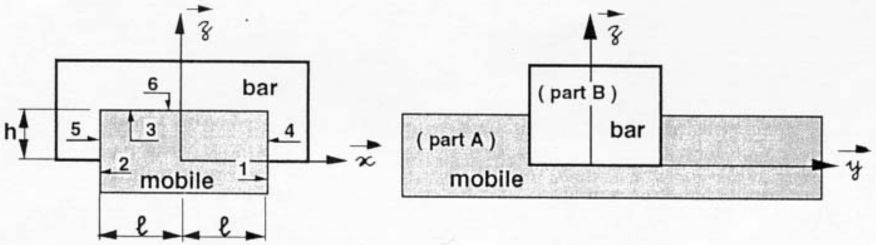
The solution for this system of linear equations of small displacements is obtained by applying the Gauss elimination method. This algorithm has been altered so as to fit the characteristics of the systems encountered with this model, i.e systems that can be simultaneously over and under determined.

As in the case of the intersection operator, there is a combination of possible positionings for each part. This is directly linked to the isostatism or the hyperstatism of the positioning of the part. The isostatic positioning of a part only yields one configuration. But the hyperstatic positioning yields a set of configurations that can then be expressed.

The consequence of variations in connected sets has thus been formally described. The operators have been defined in a formal mathematical programming language and applied to numerous cases. To illustrate part of the results obtained by the model, we are now going to apply it to an example.

5 EXAMPLE

The mechanism in figure 4 will illustrate the concepts developed. Both parts of the mechanism are joined to form a prismatic link. The possible combinations of the mobile part in relation with the bar are being searched, with the union operator of displacements. A deviation torsor is therefore associated with each surface of the mechanism, here, each plane. A small displacement torsor per part completes the modelization. The description procedure is systematic, which allows to process this link or a whole mechanism the same way; only the time of computation varies.



Each part of the mechanism is defined by:

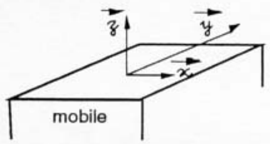
mobile = {{1,{ℓ,0,0}, {0,0,Pi}, Plan},
{2,{-ℓ,0,0}, {0,0,0}, Plan},
{3,{0,0,h}, {Pi/2,Pi/2,0}, Plan}

Coordinates point of deviation torsor — *angles between referential surface and referential mechanism*

Surface number **{1, {ℓ,0,0}, {0,0,Pi}, Plan}** *Surface nature*

Figure 4 The mechanism and its systematic description procedure

Determining the gap torsors is the result of the application of the algorithm of intersection of the deviations fields for each link plane/plane. Then, for instance, the gap torsor of the mating between horizontal planes can be:



$$\{T_{Mobile\ plane/bar\ plane}\} = \begin{Bmatrix} J_{rx} & Ind_{tx} \\ J_{ry} & Ind_{ty} \\ Ind_{rz} & J_{tz} \end{Bmatrix}_o \quad (12)$$

The assembly of the two parts is done through 3 links plane/plane.

The union operator of motions due to each mating calculates the small displacement of the mobile part in relation with the bar by elimination of undetermined values; it also calculates the varied configurations of the mechanism. These configurations, systematically listed, are due to the system of 4 independant linear equations with 7 unknowns:

$$\beta(1/A)+\beta(2/A)+\beta(4/B)+\beta(5/B)-J_{ry}(4/1)+J_{ry}(5/2)=0 \quad (13)$$

$$-\beta(2/A)-\beta(5/B)+\gamma(3/A)-\gamma(6/B)-J_{ry}(5/2)+J_{ry}(6/3)=0 \quad (14)$$

$$-\gamma(1/A)-\gamma(2/A)+\gamma(4/B)-\gamma(5/B)-J_{rz}(4/1)+J_{rz}(5/2)=0 \quad (15)$$

$$z_1J_{ry}(4/1)-z_2J_{ry}(5/2)-y_1J_{rz}(4/1)-y_2J_{rz}(5/2)-J_{tx}(4/1)+J_{tx}(5/2)+u(1/A)+...+u(5/B)=0 \quad (16)$$

there are then 3 among 7, i.e 35 potential configurations.

The configurations are directly obtained by substituting the 0 value to each set of gap components identified, so as to show contact. This contact can be full or partial as is shown in figures 5 and 6.

The algorithm lists 9 distinct configurations. A partial illustration of the results of the computation of configurations is given in figures 5 and 6. The systematic enumeration of these configurations makes sure that, when calculating the variation chains due to each configuration, one actually deals with the most unfavorable case. If all configurations do not reflect the working of the mechanism, one only has to select amidst the built set, those that are functional. For instance, the three configurations in which the horizontal plane lean is dominant.

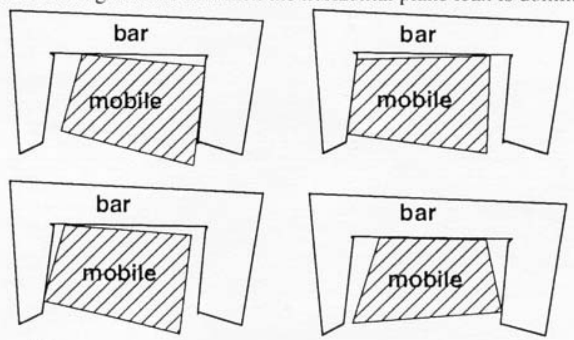


Figure 5 Examples of configurations with amplified variations (front view)

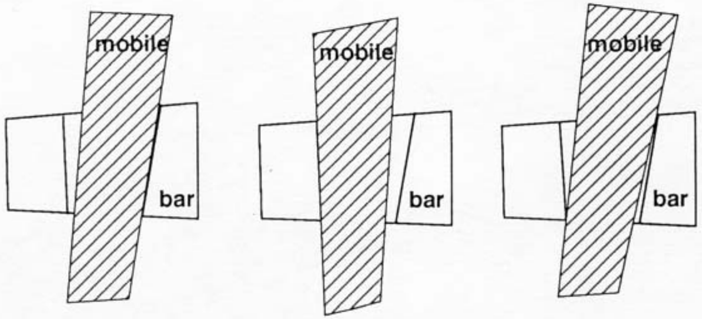


Figure 6 Examples of configurations with amplified variations (bottom view)

The application of the operator of the union of the mating displacements also shows the mobility degrees by the remaining of undetermined components in the small displacement torsors per part; here, it is the axis of the prismatic link built in the mechanism.

$$\{T_{Part\ A/Part\ B}\} = \left\{ \begin{array}{ll} -\beta(3/A)-\beta(6/B) & z2(\beta(2/A)+\beta(5/B)) - y2(\gamma(3/A)+\gamma(6/B))+u(2/A)-u(5/B) \\ \gamma(3/A)-\gamma(6/B) & \textit{Indeterminate} \\ \gamma(2/A)-\gamma(5/B) & -u(3/A)-u(6/B) \end{array} \right\}_O$$

Despite the complexity of the formal calculations at stake, the formalization of this example is systematic and does not present any difficulty. A link with a geometric modeller is actually being developed so as to suppress this phase of computation of the model.

6 CONCLUSION

The model presented here formally links the varied types of variables that variation chains represent. Our approach is founded upon a local description of the behavior of each mating. The global characteristics of the mechanism are then searched by the operator of union of small displacements. Establishing links between the varied torsors of small displacements is made possible when identifying two types of deviation variables.

The definitions of the torsors and of the operators are a first step in the formalization of tridimensional tolerance chains. These will then be obtained by the composition of the small displacement torsors of the surfaces that contain mating variations. The model, thanks to its general character, can nevertheless be thought of as applicable to manufacturing dimension chains.

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