MATHEMATICAL METHOD FOR TRIDIMENSIONAL VERIFICATION OF THE TOLERANCES OF LOCATION WITH DEGREES OF FREEDOM.

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Abstract

The paper deals with a presentation of a mathematical method for the control of composite position tolerance using CMM techniques. The location control consists in verifying that each real axis is included in the tolerance zone that can be constrained in situation by datums.The problem is written as an optimization one for which the criterion choosen takes into account the distances between the points of the axis and the frontiers of the tolerance zones. The algorithm used provides an optimum for toleranced zones limited by couples of parallel planes. The cylindrical zones is approximated by a polyhedral zone composed by couples of planes. Then, the problem to solve is linear for all the cases. The method is validated using different location specifications for a test-part.

Keywords : Location Tolerance for Holes Pattern; Measurement Techniques; Datums System

1. INTRODUCTION

For the principle specified by composite position tolerance [1] it is the true theoretical dimensions and the location tolerances that determine the position of geometrical elements (points, real axis or median plane) with respect to each other or with respect to a given datum or a given datum system. The tolerance zones are symmetrically scattered in relation to the true theoretical position.

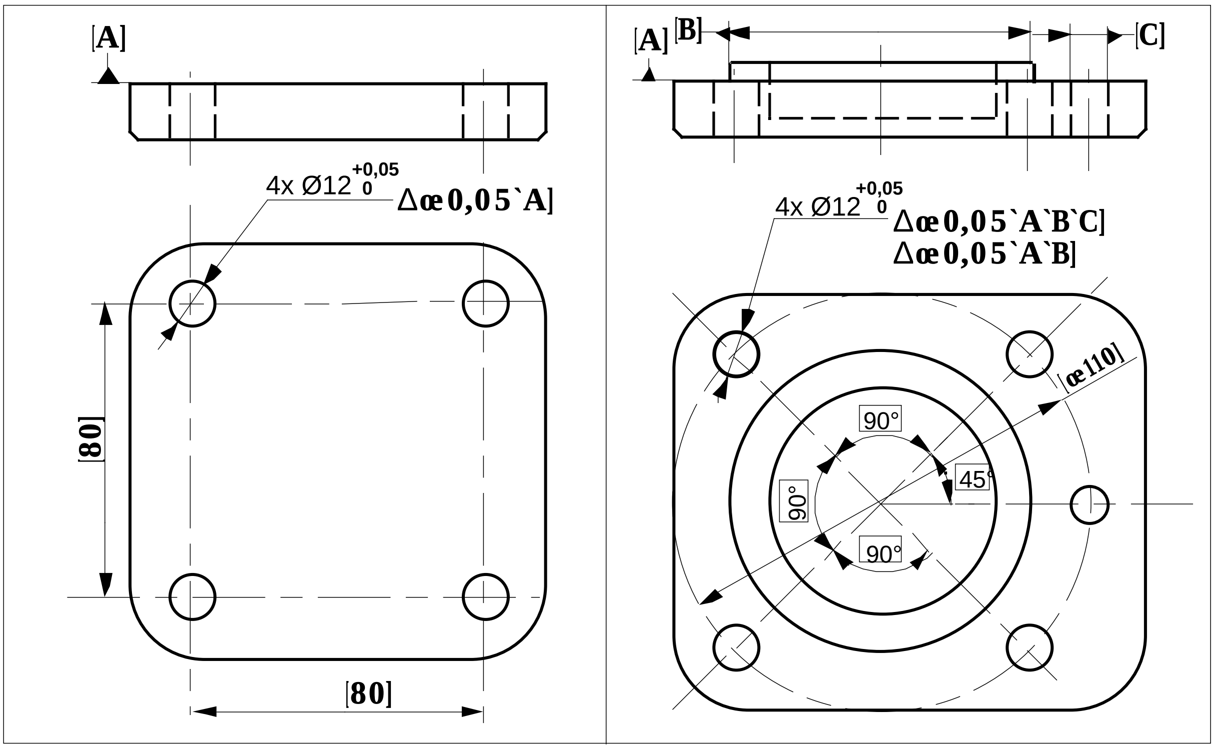

Fig.1

The tolerance zones can be of different forms which result from the relative positions of elements, the specification with datum or not, the geometrical shape of the zone (circular or unidirectional) and the specification with maximum material condition (Ⓜ) or not. They are mostly defined by a cylinder (fig. 1) or by two couples of planes, but may be spherical or projected.

In the paper, we are specially interested in the tolerances of position for a pattern of holes with cylindrical or bi-directional tolerance zones. Techniques of measure using Coordinate Measuring Machine (CMM) allow to model real surfaces by a set of points. Then, the real axis of a hole is modelled by a set of Cj points that correspond to the centers of circular sections of the hole.
Each section is normal to the axis of the theoretical cylinder fitted to the real surface and each center Cj is defined by the center of the theoretical circle fitted to the circular line. For the pattern of holes, the different real axis Ai can be modelled by different sets of points Cij.

The location control of the pattern consists in verifying that each point Cij is included in the Zi tolerance zone that can be constrained in situation by the datums. Moreover, this consists in making a relative displacement between the R frame attached to the toleranced elements with respect to the R0 frame attached to the tolerance zones Zi. Such a displacement is two-fold:

- objective of control of the specification
- objective of measure with the optimization of the location (and orientation) of the frame R with respect to a *technological* criterion.

Two problems are then raised, first the criterion choice and second the algorithm that leads to the optimal solution vis à vis this criterion [2]. Classically, the problem comes up to one of optimization, linear or not, constrained or not for which the criterion is "to minimize the maximum of distances of the points to the axis". Lehtihet [3] uses an iterative method that consists in giving for each step small displacements (x,y) to the part and a rotation (θ) supposed to be small, thus minimizing the greatest distance. This resolution of the non linear optimization leads to an approximate solution. McCann [4] solves the same problem but with a non-iterative algorithm. To find the global minimum, he realizes a suboptimization in which θ is fixed. This leads to a set of lower bound functions in rotation. Then he searches the supremum of these functions to find the global minimum. Ballu [5] presents the problem as a non linear optimization with constraints; this problem is solved by the Nelder-Mead simplex method.

In this paper we use an optimization criterion that takes into account all the distances between the points Cij and the frontiers of the tolerance zones. Then, if all the points are located within their tolerance zones, case of a "good" part, the criterion is "to maximize the smallest distance to the frontier of the tolerance zone" : max $\{\varepsilon_{jmin}\}$ (fig. 2a).

In the case where at least one point is out of the tolerance zone, i. e. "bad" part case, the criterion is "to minimize the greatest distance to the frontier of the tolerance zone" : min $\{\varepsilon_{jmax}\}$ (fig. 2b).

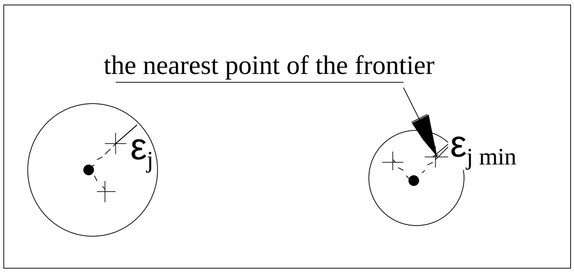

Fig. 2a

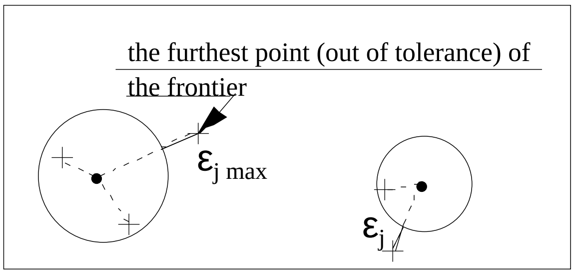

Fig. 2b

This criterion provides a realistic image of the functional side of the location tolerance for the assembly function (maximum clearance) and for the function of machine tool adjustment (control chart). Notice that in the case where all the tolerance zones have the same dimension, the criterion is identical to the classical mini-max generally used. But the interest of such a criterion is to propose a solution for specifications with different tolerance zone sizes that should soon be allowed by the norm.

In the mathematical method proposed next, we show that the problem can be written as a linear optimization one.

2. MATHEMATICAL METHOD

The problem is now to optimize, with respect to the previously described criterion, the position and the orientation of the R frame attached to the toleranced elements with respect to the R0 frame attached to the Zi tolerance zones. These zones can be constrained in situation by the datums. The column (a) of the table 1 presents the different degrees of freedom allowed by the datums.

		degrees of freedom			experimental results
		number (a)	rotations (b)	translations (c)	(d) ef (mm)
Δ œ tæ] Δætæ]	*without datum*	5	α β γ	u v	0,0298
Δœ tæ`A] Δætæ`A]	*with datum* A - plane normal to $\vec{Z}$	3	0 0 γ	u v	0,0302 case 1
	A - cylinder axis $\vec{Z}$	1	0 0 γ	0 0	
Δ œ tæ`A`B] Δætæ`A]B]	*with datums system* - 2 planes	1	0 0 0	u 0	
	- plane and cylinder	1	0 0 γ	0 0	0,0421 case 2
Δ œ tæ`A]B`C] Δætæ`A`B`C]	- 3 planes	0	0 0 0	0 0	0,0464 case 3

Table 1

Initially, the two frames are supposed to be very close. So, the relative movement of the two systems is characterized by a small displacement screw [6]:

$$D(R/R0) : \left\{ \begin{matrix} \alpha & u \\ \beta & v \\ \gamma & - \end{matrix} \right\}_{O,\ \vec{X},\ \vec{Y},\ \vec{Z}}$$

α, β, γ (small rotations) and u, v (small displacements) represent the five possible degrees of freedom. A displacement directed in the direction of the holes axis, $\vec{Z}$ has no effect. The principal cases that occurred in specifications are presented in the columns (b) and (c) of the table 1.

The mathematical method proposed is linear and can be divided in different steps.

The initialization step consists in increasing the size of the Zi tolerance zone, the radius of which is ri, with an increment ΔR. This increment which is the same for all the tolerance zones, corresponds to the greatest distance of the points to the frontier. The initial radius for each zone is : $R_i = r_i + \Delta R$.

Then, the point Cij is moved with a displacement $\vec{D}(C_{ij})$. Let ξ_{ij}^u be the deviation at the point Cij in the direction $\vec{u}$ defined by $\xi_{ij}^u = \vec{\Omega_j C_{ij}} . \vec{u}$. The optimized deviation e_{ij}^u is given by :

$$e_{ij}^u = \xi_{ij}^u - \vec{D}(C_{ij}) . \vec{u}$$

In the direction $\vec{u}$, the initial radius Ri can be decreased of a value δr if the optimized deviations e_{ij}^u verify :

$$\left| e_{ij}^u \right| \le R_i - \delta r$$

The optimal solution is obtained for the maximization of the increment δr.

This method provides an optimum for a minimal size of the tolerance zone and for tolerance zones limited by couples of parallel planes. The cylindrical zones are approximated in each section associated to a point by a polyhedral line defined by couples of parallel lines. The number of lines can be arbitrary and has no effect on the optimal solution. Actually, the optimization is made in two passes. First, each polyhedral line (defined for example by 3 couples of lines) can take any orientation (θ_k) (fig.3a). This optimization gives an approximate solution, but defines the direction $\vec{u_k}$.

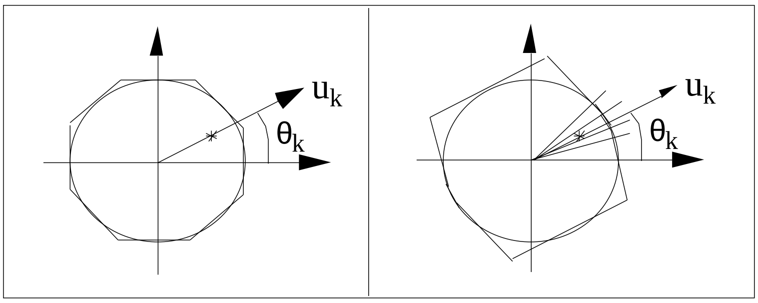

Fig.3a Fig.3b

Then, the second optimization, for which the orientation of the polyhedral lines (cleverly defined, see fig.3b) is given by $\vec{uk}$, provides the optimal solution. Tests have been done with different values of the number of lines and show that this number has really no influence on the final solution.

3. EXPERIMENTATION

The optimization problem is solved using the simplex method and the previously described algorithm is validated using the example presented on fig.1. The points Cij, representative of the four holes, are simulated using a gaussian distribution. The three cases proposed by the fig. 1 have been tested using the simulated points. The first case corresponds to a specification with "a plane normal to $\vec{Z}$ " datum and allows two translations in the plane and the rotation around $\vec{Z}$ (case 1) . For the second case, the datum system is constituted by a plane and a cylinder and the only degree of freedom allowed is the rotation around $\vec{Z}$ (case 2) . The last case does not allow any movement (case 3). Another situation, not presented in the fig. 1, has been envisaged : all the degrees of freedom are possible. For the simulations envisaged, all the tolerance zones have the same radius.

In the table 1 (column d), the value of the size of the zone holding the points after optimization is presented for the different cases. The influence of the datums is noticeable, i.e. the size is increasing with the number of constraints which is not surprising.

The fig. 4 shows the position of the points and the tolerance zones after optimization for the different cases treated.

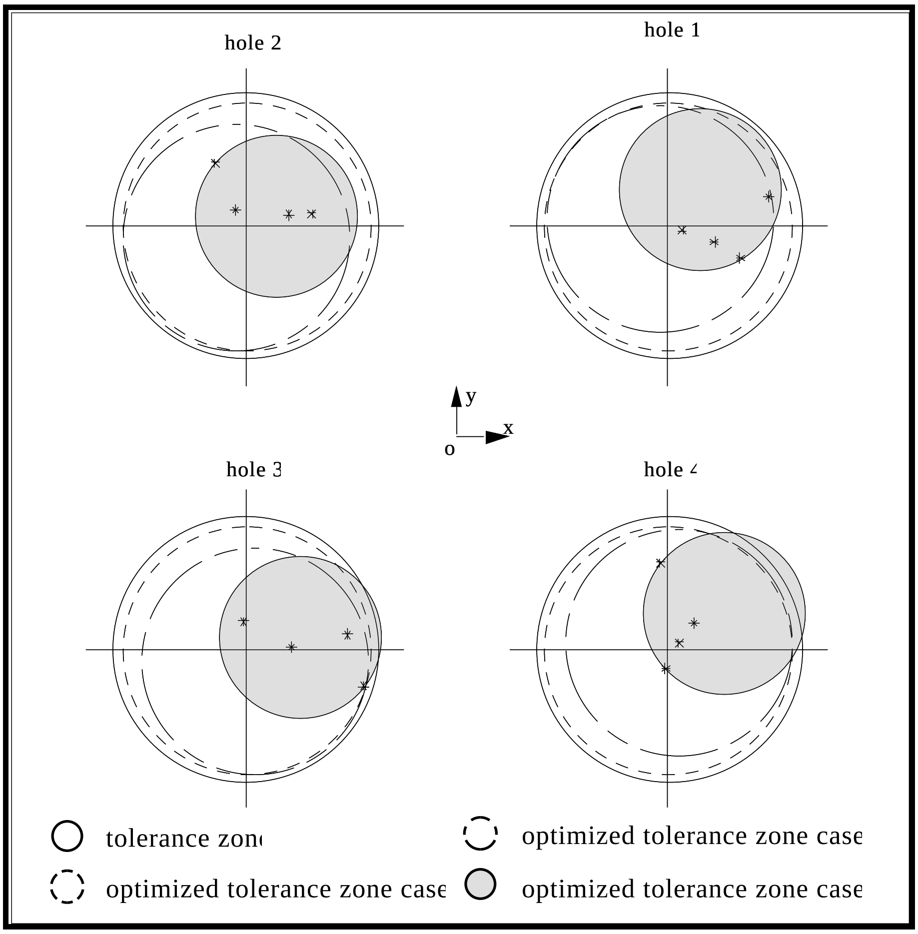

Fig.4

Whatever the case envisaged, the figure shows that the points lie inside the optimized tolerance zones. It is noticeable that the points number that defines the zone size and the zone position is at least equal to the number of optimized parameters (α, β, γ, u, v, δr). For example, the optimized tolerance zone corresponding to the case 1 should be completly defined by at least four points (0, 0, γ, u, v, δr).

4. CONCLUSIONS

This paper shows the new possibilities of computation linked to the techniques of measure using a CMM and presents a method that allows to assert whether the part is a "good" part or has to be rejected. Tolerances of location can be controlled by an optimization of the best datums system that gives the small tolerance deviations.

Moreover, with the criterion used, the treatment of composite position tolerances can be applied to tolerance zones of different sizes. Although the problem is non-linear, the linearization proposed allows to treat the case of all the shapes of tolerance zones, not only bi-directional. This provides a geometrical control of the optimization, and gives good results for short times of computation. The method, validated on simple simulated cases, has to be evaluated with data resulting from measure using a CMM.

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