Tolerance analysis in manufacturing

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The quality manufacture of a product requires integration of design on manufacturing phases within the same model of representation.
Dimensioning is one of the elements of this model ; indeed, the classical structuring of firms into partitioned departments (Taylor model) has led each department to define its own development tools. We, therefore, find a functional dimensioning method in the design department, a manufacturing dimensioning method in the manufacturing department, machine tool dimension setting methods on the shop floor as well as a dimensioning method used with measuring instruments in the quality control department. Each dimensioning method imposes constraints on the next dimensioning method down the line. This structure results in :

1) a reduction of tolerances as the product advances throughout its development cycle.
2) an irrational choice of tolerances due to lack of information on the cycle as a whole.
3) Rejection of potentially good parts due to lack of coordination between different dimensioning methods.
4) Introduction of inspection procedures during manufacturing due to the fact that tolerances have become unnecessarily restrictive.

In view of a just-in-time manufacturing philosophy implying zero defects, inspection must become an objective means for the evaluation of dimensional quality of parts, rather than being a mere discriminator between good and bad parts. This new approach can be stated as follows :
"The quality of a product cannot be simply controlled ; it must be manufactured into the product"
Under this scenario, the dimensioning and tolerancing method plays an essential role ; the tolerancing model used must be the one most compatible with the means of production and it must also minimize dimension chains relating product functional requirements to the requirements of the production method.
This approach can lead to an increase in manufacturing tolerances and enable part acceptance criteria to be set in relation to each manufacturing stage of the product.
Although several laboratories have proposed a 3-dimensional dimensioning and tolerancing model for this problem, no attempt has been made to integrate design and manufacturing phases of a product.

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In order to highlight the concept of an integrated dimensioning and tolerancing method, this paper will address only the unidirectional model.

MACHINING MODEL

We propose the Δl model [BOU-73] [BOU-75] [BOU-81] whose main principles are summarized below.
A machined workpiece is made up of surfaces (Su) obtained successively within reference frames related to the different positioning methods of the workpiece. Each positioning of the workpiece is realized by way of a reference surface (Sr) located against the fixture.
The dimension which relates two surfaces Su and Sr in the same reference frame is contained in a toleranced dimension resulting from the interaction of two Δl_i tolerance zones representative of a closed loop which includes the cutting tool/workpiece/fixture and machine tool. This loop will be modeled using two independent simulation dimensions L_i whose tolerance zones Δl_i must be greater than the minimum capability of the machine tools available (Figure 1).

Each production workpiece occupies a unique position in the reference frame attached to the machine tool.

Δl_i represents the distance between two parallel planes simulating the boundaries within which each surface Su or Sr must fall if batch production is carried out.

Δl_i is then the allowable tolerance zone of a simulation dimension Li which reprensents the position variation of a surface in a reference frame attached to the machine tool.
The causes of the Δl_i variations are assumed to be random and independent.

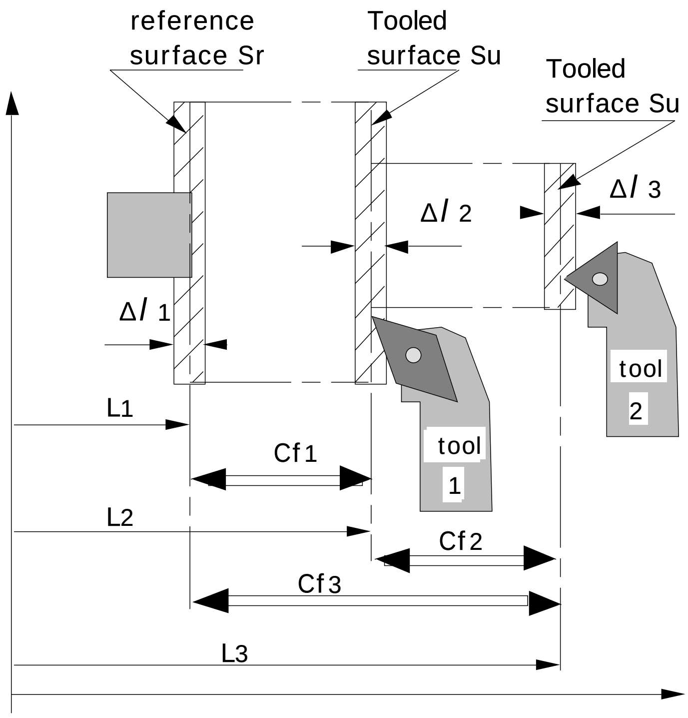


Figure 1 : machining model

P. Bourdet, J. Remy-Vincent, F. Schneider, CIRP International Working Seminar on Computer-aided Tolerancing - PENNSTATE UNIVERSITY, 16-17 Mai 1991

Consequently, each pair of surfaces Su and Sr can be related by a machined dimension Cf_j resulting from the combination of two independent simulation dimensions Li.

For a machined workpiece having 3 surfaces (fig.1) , we obtain three machined dimensions Cf_1, Cf_2 and Cf_3 of the form:

Cf_1 = resultant of $\{L_1, L_2\}$
Cf_2 = resultant of $\{L_2, L_3\}$
Cf_3 = resultant of $\{L_1, L_3\}$

The simulation dimensions are independent, it is possible to write the classical relations with centered tolerance zones :

$$\Delta Cf_1 = \Delta l_1 + \Delta l_2$$
$$\Delta Cf_2 = \Delta l_2 + \Delta l_3$$
$$\Delta Cf_3 = \Delta l_3 + \Delta l_1$$

or using maximum and minnimum limits :

$Cf_{1M} = L_{2M} - L_{1m}$ $\quad$ $Cf_{2M} = L_{3M} - L_{2m}$ $\quad$ $Cf_{3M} = L_{3M} - L_{1m}$
$Cf_{1m} = L_{2m} - L_{1M}$ $\quad$ $Cf_{2m} = L_{3m} - L_{2M}$ $\quad$ $Cf_{3m} = L_{3m} - L_{1M}$

Each tolerance zone Δl_i models the set of all possible uncertaincies ; these include a random variation Δl_{ia}, a tolerance zone for machine tool setting errors Δr (include a random variation Δra) as well as a tolerance zone for systematic errors Δs. These different errors can be determined experimentally and several authors [CAS-89] [BOU-73] have proposed various solutions. The magnitude Δ_{la} of the random part of the tolerance zone is defined by the independent criterion between the Δ_{li} and by specific experimental conditions. A three-dimensional study was proposed by [CAS-88] using face milling of a planar surface.

Machining graph :

The graph representation of this model enables a systematic solution to the selections of machining dimensions and allocation of machining tolerances.

This representation can be illustrated by a simple example with unidirectional dimensions.

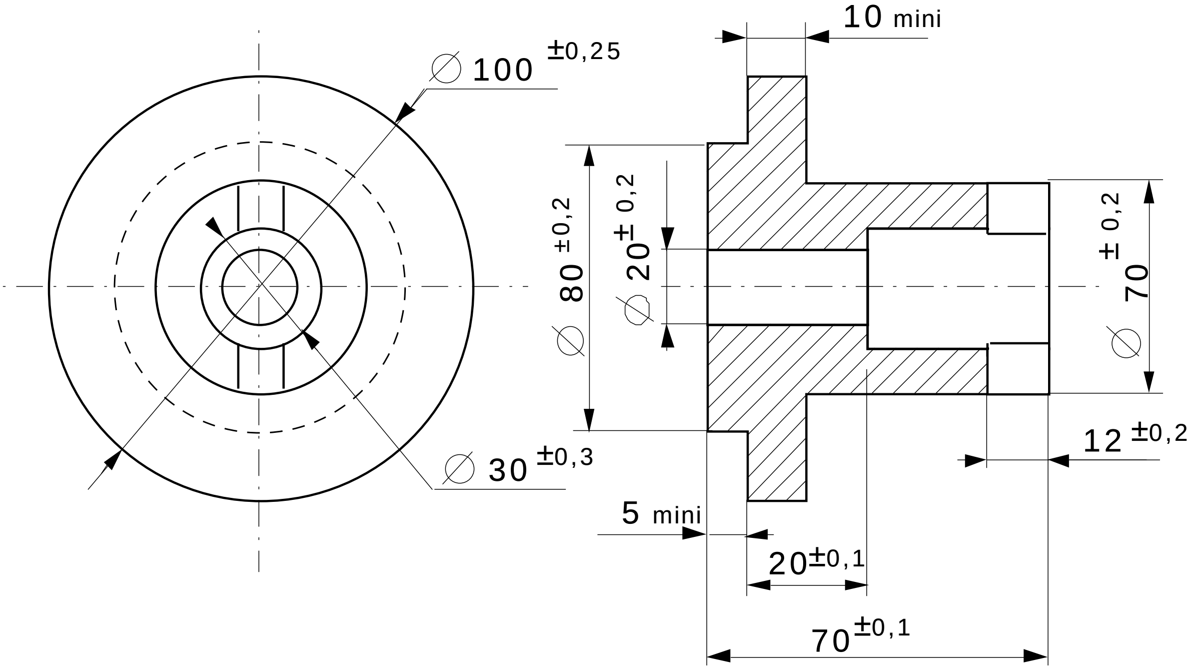


Figure 2 : BOITIER : part print

Step 1 :

The part print (fig.2) provides dimensional constraints to be satisfied. Depending on the direction to be studied, 5 dimensions must be considered : 3 bilateral dimensions (70±0,3 ; 20±0,1 ; 12±0,3) and 2 unilateral dimensions (5 mini and 10 mini).

Step 2 : Proposed Process Plan

This workpiece is machined from a forging according to the following operations :

- Lathe : Rough and finish turn diameter 100 as well as diameter 80 and its associated surfaces.
- Lathe : Rough and finish turn diameters 70, 20, 30and associated surfaces.
- Milling machine : Mill the 12 ± 0,2slot.

A fixed reference frame is attached to each operation. The different rough and finish surfaces appear on the machining graph.(fig.3).

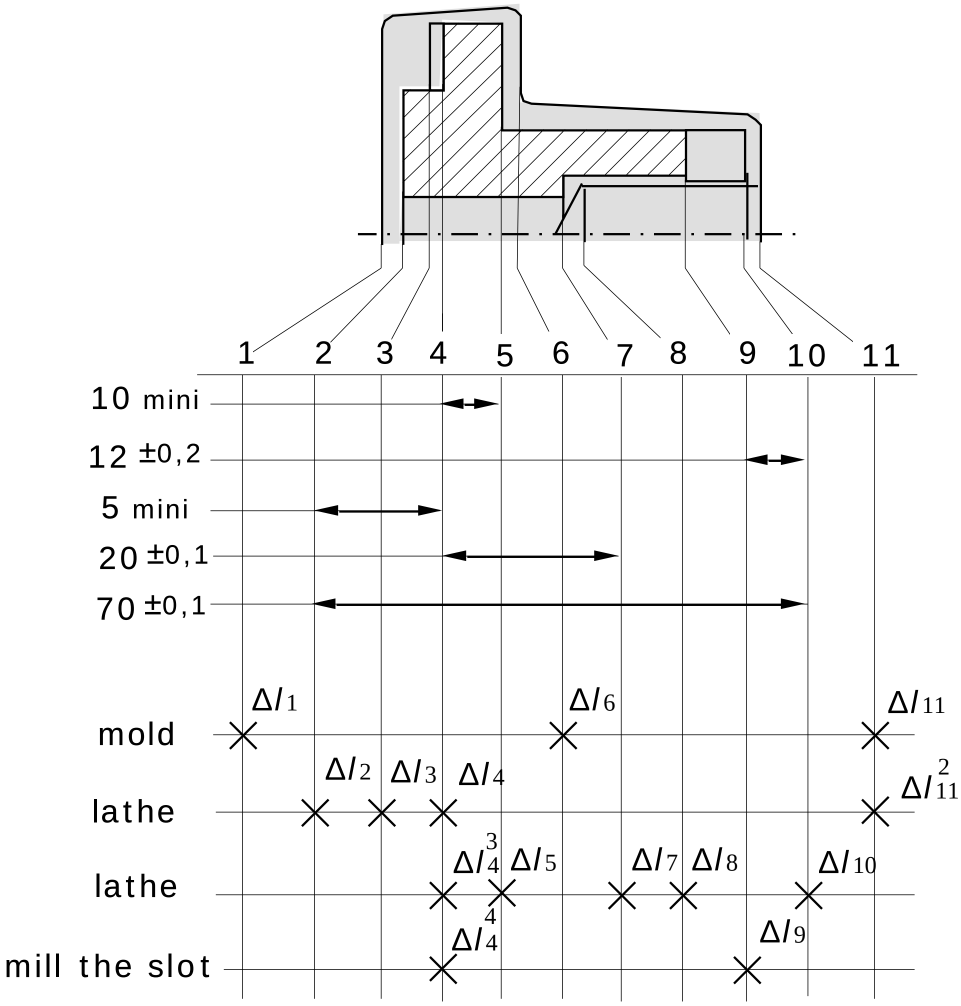


Figure 3 : Machining graph

Contraints between simulated dimensions :

Each bilateral dimension appearing on the part print generates a constraint between simulation dimensions. The 3 constraints corresponding to bilateral dimensions (70 ± 0,3 ; 20 ± 0,1 ; 12 ± 0,3) can be written directly from the graph.

(1) Variation of dimension 12 = $\Delta l_{10} + \Delta l_4^3 + \Delta l_4^4 + \Delta l_9 \leq 0{,}4$

(2) Variation of dimension 20 = $\Delta l_4^3 + \Delta l_7 \leq 0{,}2$

(3) Variation of dimension 70 = $\Delta l_2 + \Delta l_4 + \Delta l_4^3 + \Delta l_{10} \leq 0{,}2$

The system of inequalities is solved by successively maximizing the most stringent tolerances and by making sure that each Δl_i is greater than the corresponding random variation Δl_{ia}. We obtain :

$\Delta l_2 = 0{,}05$ $\Delta l_4^3 = 0{,}05$ $\Delta l_7 = 0{,}15$ $\Delta l_9 = 0{,}15$

$\Delta l_4 = 0{,}05$ $\Delta l_4^4 = 0{,}15$ $\Delta l_{10} = 0{,}05$

Tolerances which are not imposed by bilateral dimensions specified on the part print can be chosen in order to facilitate manufacturing.

$\Delta l_3 = 0{,}15$ $\Delta l_5 = 0{,}15$ $\Delta l_{11}^2 = 0{,}3$ $\Delta l_8 = 0{,}15$

The part print requires two unilateral dimensions : 10 mini and 5 mini. In machining, it is good practice to insure that minimum stock removal amounts of 2 mini for roughing and 0,2 mini for finishing are available for each operation. These requirements enable us to write directly from the graph the following equations :

(4) Variation of dimension 10mini = $\Delta l_4^3 + \Delta l_5 = 0{,}2$

(5) Variation of dimension 5mini = $\Delta l_2 + \Delta l_4 = 0{,}1$

Variability of roughing pass stock removal :

(6) between surfaces 1 and 2 : $\Delta l_1 + \Delta l_{11} + \Delta l_{11}^2 + \Delta l_2 = 2{,}35$

(7) between surfaces 5 and 6 = $\Delta l_6 + \Delta l_{11} + \Delta l_{11}^2 + \Delta l_4 + \Delta l_4^3 + \Delta l_5 = 2{,}55$

(8) between surfaces 10 and 11 = $\Delta l_{10} + \Delta l_4^3 + \Delta l_4 + \Delta l_{11}^2 = 0{,}45$

Variability of finishing pass stock removal :

(9) between surfaces 3 and 4 = $\Delta l_3 + \Delta l_4 = 0{,}2$

(10) between surfaces 7 and 8 = $\Delta l_7 + \Delta l_8 = 0{,}3$

Manufacturing dimensions :

The 10 previous relations simulate production operations and one can state that part print requirements will be satisfied if all 10 relations hold true during production.

These 10 relations, therefore, impose raw material dimensions as well as dimensions to be respected during each operation. The only requirement is to verify the sum of the Δl_i relevant to each operation of the process plan.

For example, relation (7) $\Delta l_6 + \Delta l_{11} + \Delta l_{11}^2 + \Delta l_4 + \Delta l_4^3 + \Delta l_5$ imposes the following dimensions :

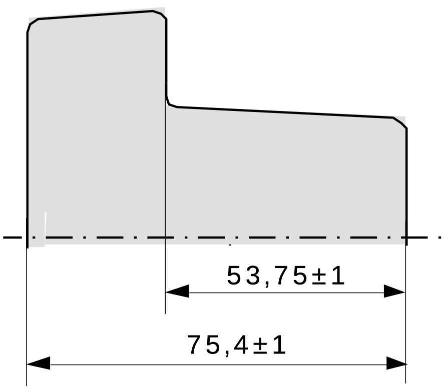
$\Delta l_6 + \Delta l_{11}$ for the dimension between surfaces 6 and 11 of the raw preform.

$\Delta l^{2}_{11} + \Delta l_{4}$ for the dimension between surfaces 11 and 4 machined in the first turning operation.

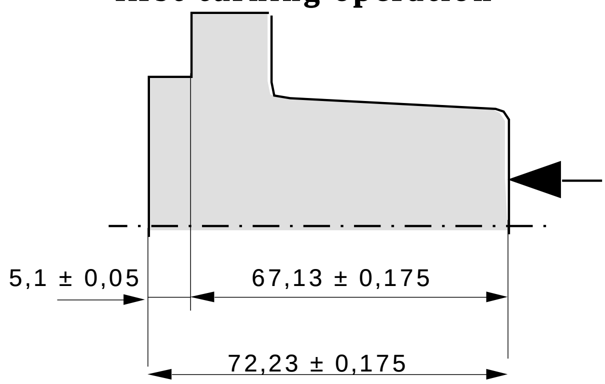
$\Delta l^{3}_{4} + \Delta l_{5}$ for the dimension between surfaces 4 and 5 machined in the second turning opération.

After computing mean dimensions, the following machined dimensions are obtained :

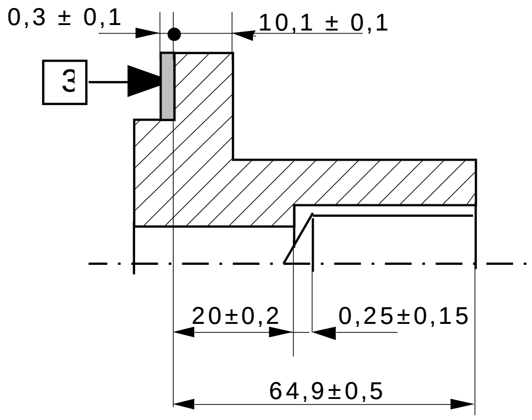
Dimensions of raw preform



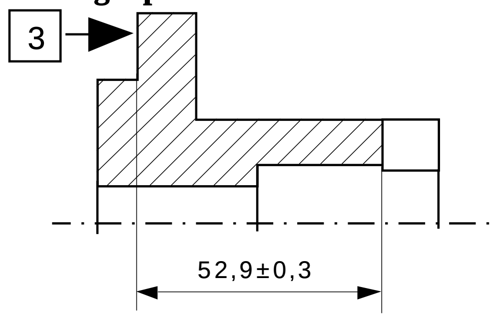
Dimensions
first turning operation



Dimensions
second turning operation



Dimensions
milling operation



Some observations regarding machine tool setting :

Machined dimensions can be used to check if a workpiece machined during an operation (independent of other operations of the process plan) will satisfy part print requirements.

Machined dimensions are modeled by combinations of simulation dimensions Li. Within each operation, simulation dimensions enable statistical monitoring of machine tool setting conditions. For example, considering the firs turning operation, every workpiece in the batch will be acceptable if surfaces (S_2-S_4); (S_4,S_{11}) and (S_2-S_{11}) are contained within machined dimensions ($5{,}1 \pm 0{,}05$); ($67{,}13 \pm 0{,}175$); ($72{,}23 \pm 0{,}175$). A statistical analysis of batch dimensions during production will allow determination of random variations Δl_{ia} and their position in a fixed reference frame.
The following drawing (fig.4) shows that an improperly set machine tool can produce acceptable workpieces provided, for example, that two adjacent dimensions are realized at their maximum condition.

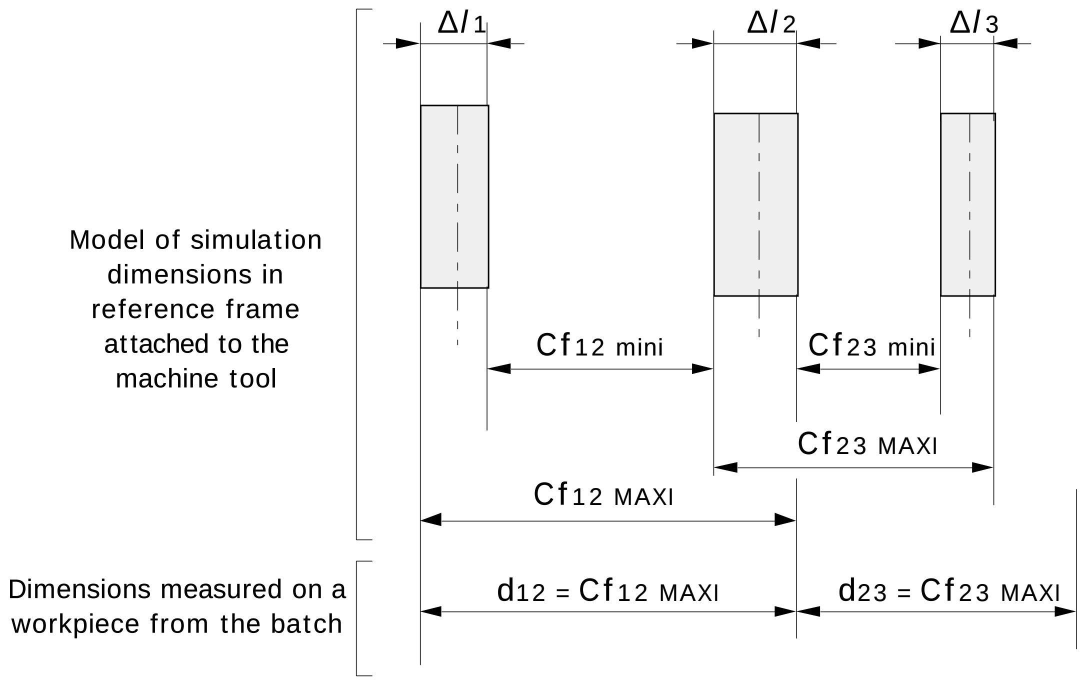


Figure 4 : An improplely set machine tool and an acceptable worpiece

MODELING THE FUNCTIONS OF A MECHANISM

The conception of a product takes place around the functions this product must fulfill. A preliminary study of the product expresses these functions in terms of functional conditions.

Compliance with functional conditions requires an identification of relevant workpieces and their associated geometric constraints.

In the unidirectional case, functional conditions will be modeled by distances bounded by surfaces having the same orientation

As an example, functional requirements for the position of the roller of figure 5 can be translated into functional condition e expressed by

$$p_{mini} \leq e \leq p_{MAXI}$$

Assembly requirements for the roller 3 and parts 2 and 1 can be expressed by two functional conditions f and g such that $f \geq k_{mini}$ and $g \geq k_{mini}$

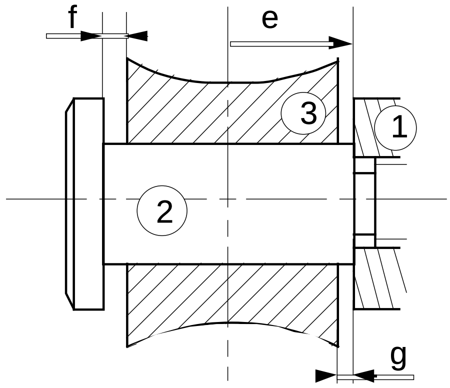


Figure 5 : Conveyor belt roller

The behavior of a mechanism manufactured in bath production is modeled by the "functional flux of constraints". This model highlights geometric relations between various parts of the mechanism. Toward this end, a reference frame is attached to each part of the mechanism and the relative positions of the reference frames are determined by considering the possibilities of contact between surfaces. It is then possible to write the "functional flux of constraints" which translate different functional conditions. Several choices of "functional flux" can exist for one functional condition, they must all be verified.

Graph of the mechanism :

The constraints are determined with the aid of a graph (fig. 6). A mechanism graph must be constructed for each position of the mechanism.

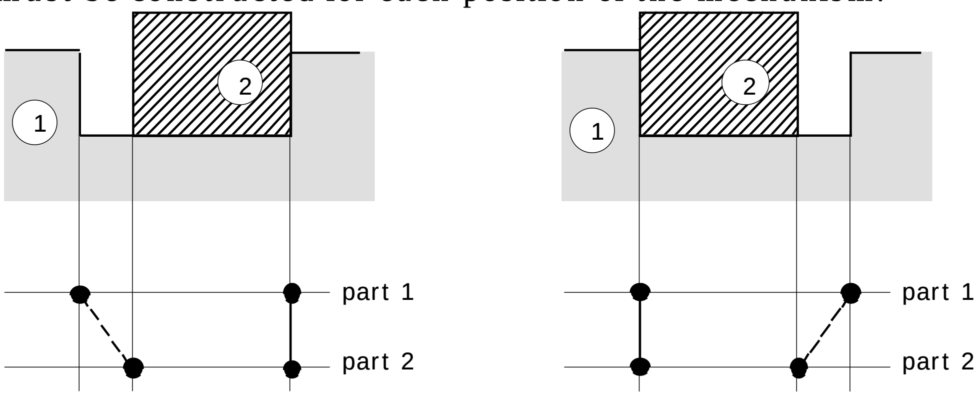


Figure 6 : mechanism graph

A reference frame (horizontal line) is attached to each part of the mechanism. In this reference frame, part surfaces are represented as points corresponding to the projection of the surface on the reference frame.

The relative positions of reference frames are determined according to the possibilities of contact between parts : a vertical line linking two points on two different parts characterizes this contact.
In order to take into account the bilateral nature of contacts and to limit the number of graphs, DEWULF and COGIBUS (DEW-76) proposed a simplified graph where the mechanism is represented without clearances (figure. 7).

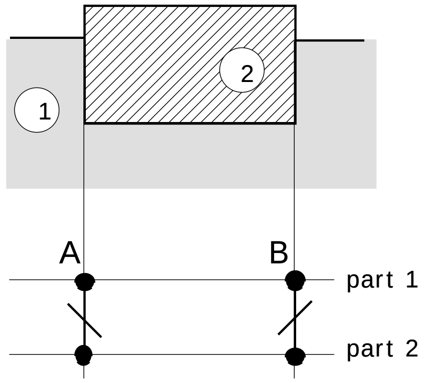
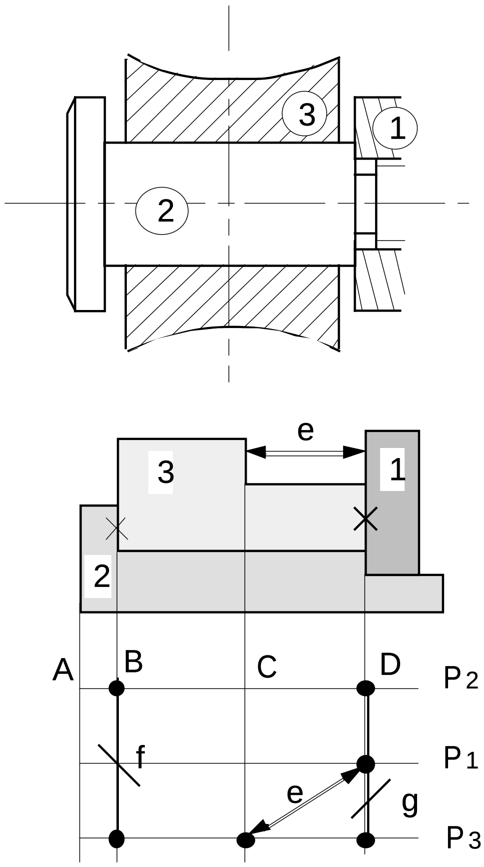


Figure 7 : simplified graph mechanism

All parts with surfaces associated with a bilateral contact are represented in a state of contact.
An inclined segment indicates the sens of contact between surfaces which permit a transition from the containing part to the contained part.
This graph allows an expert to establish relations which translate functional conditions between the dual points.
Going back to the example of a roller, the mechanism graph is given in figure 8.
The "e" dimension is associated with the functional condition expressing roller position which is limited by pMAXI and pmini. The f and g dimensions limited by kmini are two functional conditions expressing assembly requirement of roller 3 between parts 2 and 1.
The "e" functional condition demands the study of two chains which correspond to the two posible positions of the mechanism :

to e_1 is associated
$C_3D_1 = CD_3 (+D_3D_1)$
to e_2 is associated
$C_3D_1 = BD_2 - BC_3 (+D_2D_1)$



conveyor belt roller
Figure 8 : mechanism graph

to assembly condition f or g is associated : $B_2B_3 = D_1D_2 = BD_2 - BD_3$

Applying the functional condition to the mechanism under consideration leads to the following inequalities :

e1 : $C_3D_1 = CD_3 \ (+D_3D_1) \geq p$ mini
e2 : $C_3D_1 = BD_2 - BC_3 \ (+D_2D_1) \leq p$MAXI
f ou g : $B_2B_3 = D_1D_2 = BD_2 - BD_3 \geq k$ mini

Since dimensioning of the 3 parts must satisfy the above inequalities, many dimensioning solutions are possible. If dimensioning is left to the design expert, lack of manufacturing information will lead this expert to make an arbitrary and perhaps unduly restrictive choice vis à vis production capabilities.
It is possible to integrate the modeling of production and design phases [BOU-81],[REM-91]

A GLOBAL APPROACH TO DIMENSIONING. optimization of tolerancing

In this case, the choice of dimensions is defined by functional conditions and by dimensions simulating the manufacturing process. There will therefore be a optimal dimensioning distinct to each projected manufacturing process.
A single dimensioning graph combining the mechanism graph and the part manufacturing graphs corresponds to each process. Immediate optimization results and we obtain functional and machining prints.

The choice between various dimensioning and tolerancing solutions can be done for according to several criteria : by maximizing tolerances, by using processes imposed by the available machining cells, by minimizing part weight, by minimizing total mechanism cost... etc...

As an example, a study of dimensioning of parts 3 (roller) and 2 (shaft) of the belt roller will illustrate the procedure.

Two roller manufacturing processes are considered :

- Machining of surfaces B, C and D in one operation.

- Machining in two operations : operation one machines the roller to length (surfaces B and D) while operation two locates the part on surface B and machines the form positioned by surface C.

A dimensioning graph is established for every machining possibility (fig. 9)

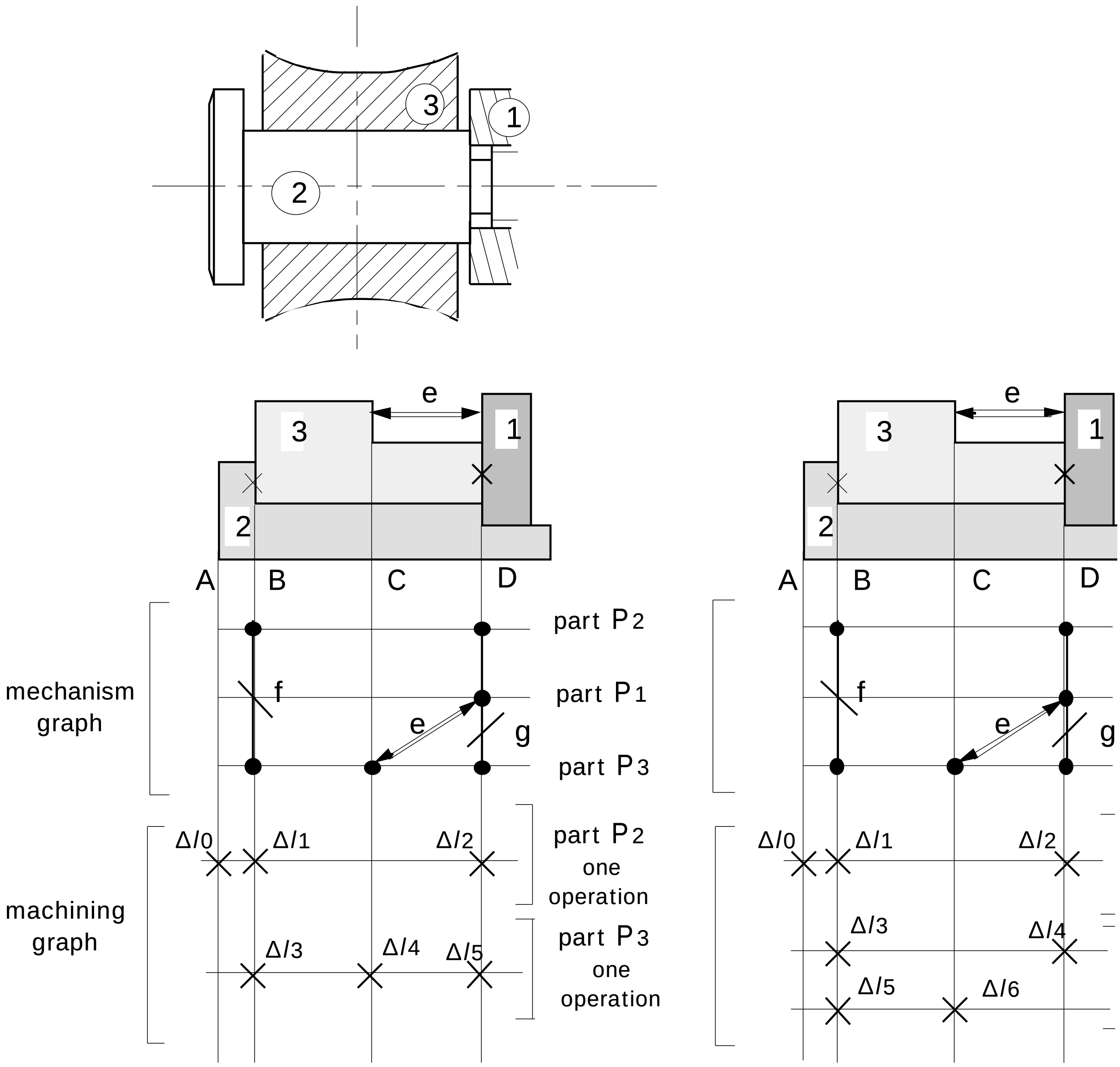


Figure 9 : Dimensioning graphs corresponding two fabrication method
Constraints deduced from functional conditions.

Constraints deduced from functional conditions.

(1) $f_{mini} = g_{mini} = BD2_{MAXI} - BD3_{mini} \geq k_{mini}$

(2) $e1_{mini} = CD3_{mini} \geq p_{mini}$

(3) $e2_{mini} = BD2_{MAXI} - BC3_{mini} \leq p_{MAXI}$

These equations are true if the dimensions are independent.

Constraints for geometric simulation of the processes

The dimensions mini and maxi manufacturing are given according to mean dimensions and tolerances Δl_i .

For example : $BD2_{MAXI} = BD2_{mean} + \Delta BD2 / 2$

$BC3_{mini} = BC3_{mean} - \Delta BC3 / 2$

For the first machining method we obtain :

$f_{mini} = g_{mini} = BD2_{mean} + (\Delta l1+\Delta l2)/2 - (BD3_{mean} - (\Delta l3+\Delta l5)/2) \geq k_{mini}$

therefore we obtain :

(1) $f_{mini} = g_{mini} = BD2_{mean} - BD3_{mean} + (\Delta l1+\Delta l2+\Delta l3+\Delta l5)/2 \geq k_{mini}$

(2) $e1_{mini} = CD3_{mean} - (\Delta l4+\Delta l5)/2 \geq p_{mini}$

(3) $e2_{mini} = BD2_{mean} - BC3_{mean} + (\Delta l1+\Delta l2+\Delta l3+\Delta l4)/2 \leq p_{MAXI}$

For the second machining method we obtain.

(4) $f_{mini} = g_{mini} = BD2_{mean} - BD3_{mean} + (\Delta l1+\Delta l2+\Delta l3+\Delta l4)/2 \geq k_{mini}$

(5) $e1_{mini} = CD3_{mean} - (\Delta l3+\Delta l4+\Delta l5+\Delta l6))/2 \geq p_{mini}$

(6) $e2_{mini} = BD2_{mean} - BC3_{mean} + (\Delta l1+\Delta l2+\Delta l5+\Delta l6)/2 \leq p_{MAXI}$

As a first approximation, if all Δl_i manufacturing tolerances represent the same level of difficulty say Δl_o then a different Δl_o value is obtained for each manufacturing method.

First method :

(1) $f_{mini} = g_{mini} = BD2_{mean} - BD3_{mean} + 2.\Delta l_o$

(2) $e1_{mini} = CD3_{mean} - \Delta l_o$

(3) $e2_{mini} = BD2_{mean} - BC3_{mean} + 2\Delta l_o$

soit $\Delta l_{\mathbf{o}} = (e2_{mini} - e1_{mini} - g_{mini})/3 \leq p_{MAXI} - p_{mini} - k_{mini} /3$

Second method :

(4) $f_{mini} = g_{mini} = BD2_{mean} - BD3_{mean} + 2.\Delta l_o$

(5) $e1_{mini} = CD3_{mean} - 2\Delta l_o$

(6) $e2_{mini} = BD2_{mean} - BC3_{mean} + 2\Delta l_o$

soit $\Delta l_{\mathbf{o}} = (e2_{mini}-e1_{mini} - g_{mini})/4 \leq p_{MAXI} - p_{mini} - k_{mini} /4$

This simple example shows the influence of the manufacturing method on optimal values of admissible variations.

The different dimensioning schemes (functional and production) are then automatically defined. In order to simplify the example, consider this application where k_{mini} = 0,2mm ; (p_{MAXI} - p_{mini}) = 1,4mm ; BD_{3mean} = 100mm ; BC_{3mean} = 50mm ; CD_{3mean} =50mm

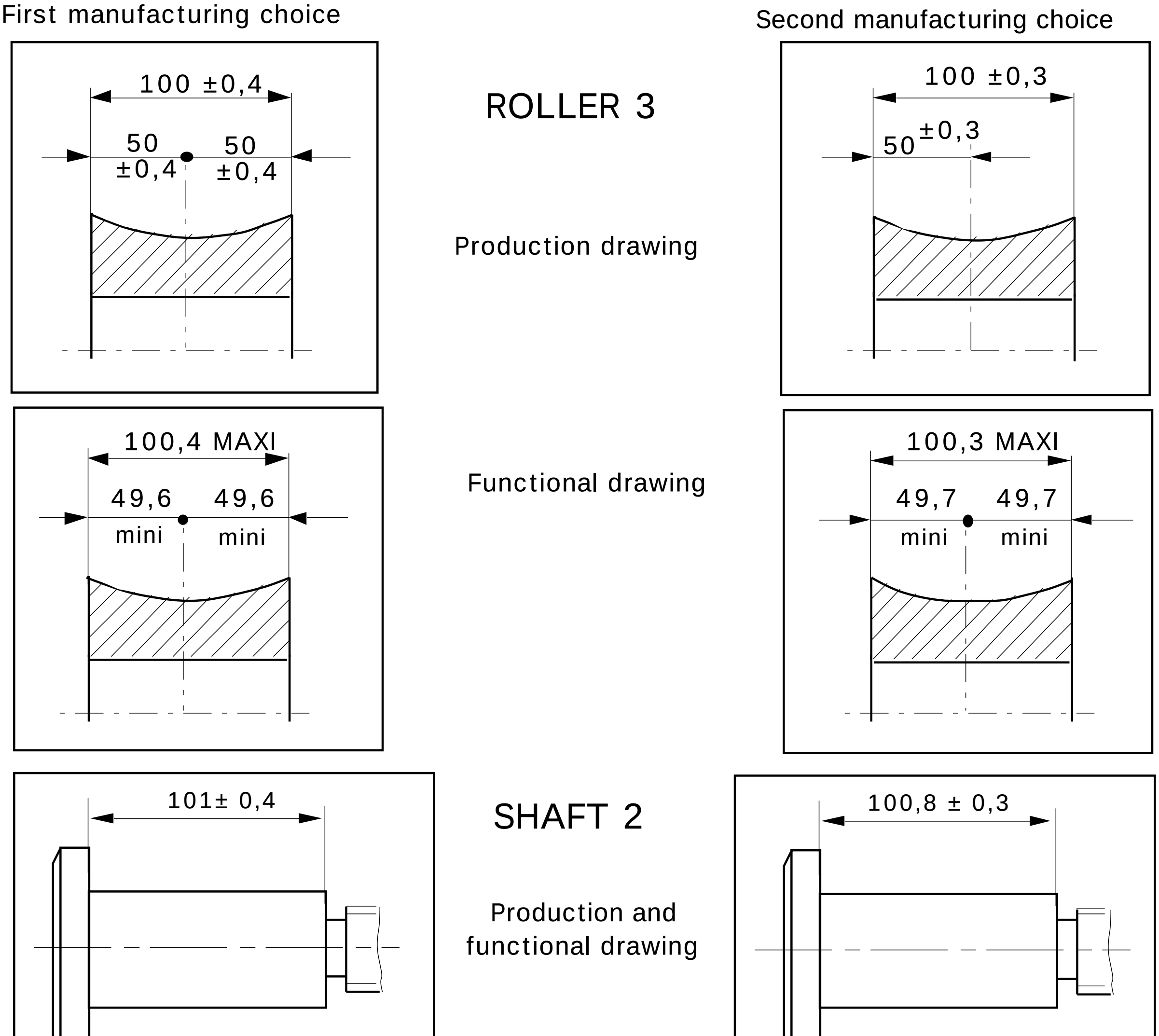


Figure : 10

Note that in this example, the dimensioning of shaft 2 is dependent of the manufacturing method of roller 3.

The value of Δl_o and therefore the manufacturing process defines the site of different dimensions of functional and production drawing. On the other hand, the position of dimensions on the functional drawing is independent of the processes.

CONCLUSION

We have just shown that it is possible to optimize the solution to the problem of dimensioning a given mechanism.
This solution is the result of a systematic procedure which is no longer arbitrary. It takes into account three important facts : the function of the mechanism and its behavior resulting from its design and manufacturing method. The basic idea is that only those people responsible for the creation phase of the parts of a mechanism must stipulate their own geometric constraints. Tolerancing optimization is then carried out within the same representation model.
The final specifications have the advantage of being unique and optimizing the objectives and requirements of both design and manufacturing.
The concepts and procedures developed here for the 1 dimensional case can be extended to 3 dimensions.

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