The Processing of Measured Points in Coordinates Metrology in Agreement with the Definition of Standardized Specifications

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Increasingly, coordinate measurement techniques are used to ensure that mechanical parts conform to their geometric specifications. The analysis of the possibilities of the software in use brought to light an important number of tools, which however are not always adapted to the problem which is to be solved. Generally these tools only give one approximate answer to the problem set out by the verification of the dimensional and geometrical specifications.

An indept study of the standards lead to the presentation, in this paper, of the problem typology, the two aspects of the verification of the specifications (measurement and control) will be presented and also the implication which these have on the solution algorithms. Finally, an attempt to define new processes giving the true value of the size to be verified is proposed. These proposals will be illustrated by an example, and the numerical results will be analysed and compared.

Key words : Dimensional Metrology, Coordinate Measuring Machines, Measurement Standards

1.INTRODUCTION

The geometric form of parts is determined by the geometric surfaces which delimit it. In metrology it is necessary to distinguish the real form of the parts by the perfect geometric form of the parts. A real geometric surface is defined by the set of all the points belonging to the material-environment interface while a perfect geometric surface is defined by a set of points linked by known mathematical relationships.

To ensure the functional role which each of the real surfaces must fulfill, it is dimensioned so that it allows the definition of all the deviations with respect to the perfect geometric form of the part. This dimensioning, which is a standardised language, expresses the requirements of a set of dimensional geometrical specifications.

The analysis of this language brings to light essentially two types of elements specified, *toleranced elements* and *datum elements*. The toleranced elements are real lines and real geometric surfaces, the datum elements are perfect lines and perfect geometric surfaces.

The toleranced elements can be either :

- basic elements (plane, cylinder, cone...)
- or groups of elements (holed structure for example)

The datum elements can be either :

- single data (plane, cylinder, cone...)
- or datum systems (a combinaison of single data)

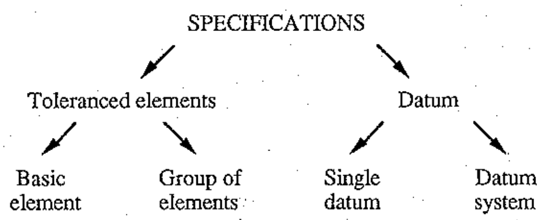

Figure 1

The interpretation of each specification leads to the construction of a geometric model which will be associated with the toleranced elements. This model consists of one or several perfect geometric elements constrained in both position and orientation by the intrinsic model definition parameters.

Three cases considered to associate this model with the toleranced elements follow :

FIRST CASE : The verification of a specification without a datum

- if the specification without a datum applies to a basic toleranced element, then the model defines the tolerance zone characterized by a parameter t.
- if the specification whithout a datum applies to a group of toleranced elements, then the model allows to position the tolerance zones characterized by a parameter t.

In this case, the specification will be verified if the association of the model with the toleranced elements allows the toleranced elements to fit into the toleranced zone(s). It is therefore a question of defining the relative position of the model with respect to the toleranced elements.

SECOND CASE : The verification of a specification with a datum

If the specification with either a single datum or a datum system applies to a basic toleranced element or to a group of toleranced elements, then the model finds itself constrained in position and orientation by the datum element(s).

The specification will be verified if the association of the model with the toleranced elements allows the toleranced elements to fit into the tolerance zone(s).

THIRD CASE : Construction of a datum

- for the construction of a single datum, the association of the model characterized by a variable d with the real reference surface is such that this variable be a maximum or a minimum.
- for the construction of a datum system, the model of the second element of a system with two data is constrained in position and orientation by the model of the first reference element already associated. The association of the second model characterized by a variable d with the real datum surface is such that this variable be a maximum or a minimum.

For a system with three data, the model of the third element finds itself constrained by the two previous models already associated.

These three cases presented above represent the problem to be solved by coordinate metrology.

2.COMPARISON OF THE CALCULATION METHODS

In coordinate metrology, a real geometric surface is only given by a finite set of points (points measured). These points are obtained by calculation. Since the coordinates picked up by the coordinate measuring machine are those of the centre of the probe (points captured), it is necessary, in the first place, to define the direction of a best-fit perfect surface passing throught the points captured (associated surface) and in the second place to make each captured point correspond to a measured point (the intersection of the normal to the associated surface with the calibrated sphere of the probe). The method of association chosen is the most often retained to define the best-fit real surface (equivalent real surface).

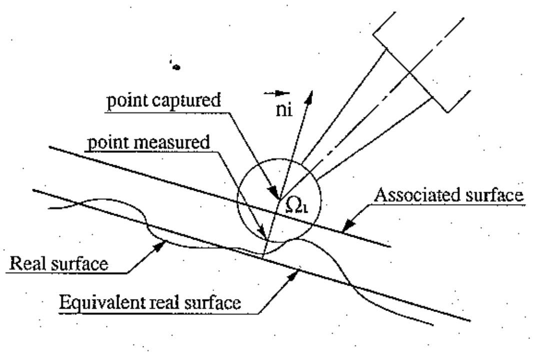

Figure 2

In the remaining part of this paper the hypothesis is taken that the measured points belong to the real surfaces and that they are representative of the real surfaces vis à vis the interpretations of the standards and the calculation methods used.

The calculation methods used in coordinate metrology software packages define, according to the least-squares criterion, the single datum and essentially respond to the need to garantee, by an estimate of the value of the magnitude characterizing the tolerance zone, whether or not the basic toleranced element is effectively in the specified tolerance zone. The methods developped are adapted to industrial control and their comparison necessitates a global approach to the problem where the cost of the methods must largely be taken into account (number of measurement points, programming time, calculation time) vis à vis the correctness of the result obtained. The calculation methods, in this case, are difficult to compare. With the measurement, on the other hand, the optimum value of the magnitude which characterizes the tolerance zone is sought, therefore it is a matter of guaranteeing that the result follows an optimisation function. If this fonction is an expression of the standardization, the comparison between the different calculation methods becomes possible by a simple comparison of the different values of the standardised magnitude obtained from the same set of measurement points.

3.THE SPECIFICATION PROCESSING PROCEDURE IN COMMON USE

3.1.Process

At the moment defects are determined by a process which has three main stages : simple model association, geometric construction, and finally characteristic determination.

Simple model association by the least-squares criterion

The majority of software programmes are based on the concept of simple model associations with a set of points. All software programmes know how to define a straight line, a plane, a cylinder or a sphere by the least-squares method.

Occasionally the possibilities can be streched to include elements which are more complex but whose form is still a basic geometric element.

This type of association gives a solution to the following model and function to be minimised :

Model :
Element :
- *a basic element A*

Function to be minimised :
- *the sum of the distances squared of the points measured on A*

Geometric construction

In this, the second stage, numerous geometric constructions, using previous elements defined by association or construction, are possible. These constructions allow the definition of new elements or coordinate systems linked to the part.

Characteristic determination

The final result of this process is a parameter, intrinsic to one element (a diameter for example) or between two (i.e. an angle or a distance).

3.2.Example

If we consider the localization examples, illustrated in figure 3, we notice that they will be, in general, processed in the same way wether or not there is a datum. The tools which are presently available make it necessary to give a coordinate system to the part, an identical coordinate system whatever the datum indicated by the specification.

The processing procedure is threfore the following :

- the determination of the plane A, associated with the points measured on the corresponding surface of the part
- the determination of the cylinder B, associated with the points measured on the corresponding surface of the part
- the construction of the straight line C perpendicular to A and passing through the intersection of A with the axis of B

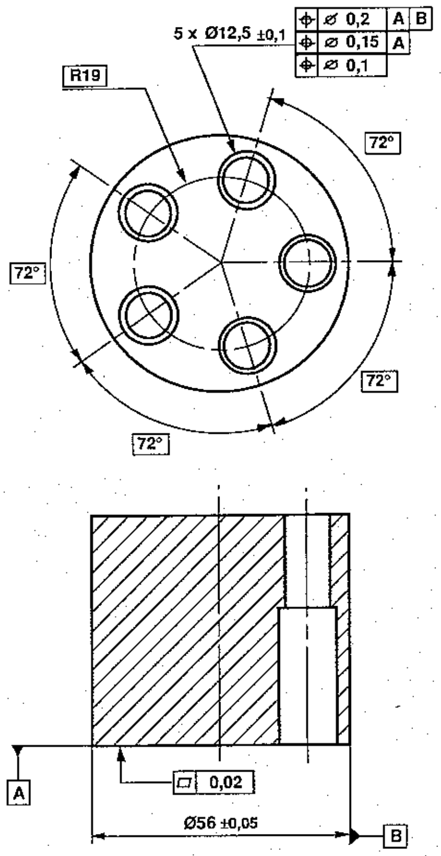

Figure 3

- the construction of the straight lines D1, D2, D3, D4 and D5 at regular intervals on the cylinder centred around the C axis of diameter 38mm, and such that the line D1 passes through one of the section centres
- the determination of the set of distances from the centres of the section to their corresponding lines
- the value retained is the largest of these distances

3.3.Conclusion

It is by his savoir-faire, by the more or less judicious choice of geometric constructions and calculated characteristics, that the operator obtains a value which is marred by mistakes.

4 ATTEMPS TO DEFINE NEW PROCESSES WHICH GIVES THE TRUE VALUE

4.1.Concepts

In the preceeding section it was seen that the control or measurement of defects is, in general, carried out from only one basic concept, the association of a basic geometric element (straight line, plane, circle, cylinder, cone) with a set of points measured, in accordance with the least-squares criterion. It was also seen that this basic concept does not allow the rigurous resolution of a given problem, but an approximate one.

In this paragraph the three concepts, which must be introduced to solve this problem, are presented.

Concept 1 : the association criterion

To construct a datum or to determine a geometrical defect, the standards never use the least-squares criterion but use Chebyshev's criterion instead. The maximum or minimum of a set of distances, to construct the datum or to measure a defect, must be optimised. Several papers [3][4][5][6] directed their research towards methods of resolution according to Chebyshev's criterion. This concept is applied in the option "Balancing of a set of circles" of the software package UMESS, since the operator has the choice of the criterion.

Example

To construct the datum A in the cases of localization with datum we are going to find the solution to the following model :

Model :
Element :
- *a plane A*
Function to be minimised :
- *the maximum of the distances between the points and the plane P*

Latter it is sufficient to translate the plane P to obtain the plane A, exterior to the material and tangent to the points meseared.

Concept 2 : Association with a complex model unconstrained by any datum

The second important concept is the association of complex models with a set of measured points, and not just the basic models. This concept allows the case of a toleranced group of elements to be processed. Complex models are therefore defined as a composition of single models for example two parallel planes or several parallel straight lines. Thus, the optimisations carried out globally for the set of elements [1].

This concept, limited to problems in the plane only, is also applied in the software package UMESS.

Example

Taking up again the previous example, it is also possible to directly determine A defined by the following model :

Model
Elements :
- *2 planes A and P*
Internal constraints :
- *the planes A and P are parallel*
Relative position of the real points with respect to the model :
- *A is exterior to the material*
- *the corresponding points measured are contained between A and P*
Function to be minimised
- *Distance between A and P*

Example

The localization without a datum is defined with the aid of the following model :

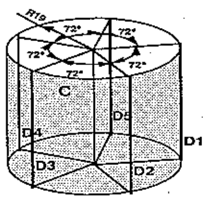

Figure 4

Model :
Elements :
- *5 straight lines Di*
- *a cylinder C*
Internal constraints :
- *the diameter of the cylinder C is 38mm*
- *the lines Di are at regular intervals on C*
Function to be minimised :
- *the sum of the squares of the distances between the centres of the sections to the corresponding line Di (least-squares criterion)*

or alternatively
- *the maximum of the distances between the centres of the sections to the corresponding lines Di (Chebyshev's criterion)*

Concept 3 : Association of a constrained model by a datum

In order to be able to take into account the elements which serve as a datum to a model (in the case of the measurement of the defect of a specification with a datum or in the case of the construction of the minor elements of a datum system) it is necessary to be able to impose constraints to the model with respect to the datum elements.

Example

To construct the reference cylinder in the case of the localization with respect to A and B, it must be imposed that the cylinder associated with the measured points remains perpendicular to the plane A already constructed in a previous stage :

Model :
Elements :
- *a cylinder B*
Constraints on the reference elements :
- *the cylinder B is perpendicular to the plane A*
- *the measured points are interior of the the cylinder B*
Function to be minimised :
- *the diameter of B*

Example

To measure the localization defect with respect to A, the plane A having been already constructed, it is necessary to impose on the cylinder, defining the relative position of the 5 lines, the constraint that it must be perpendicular to A.

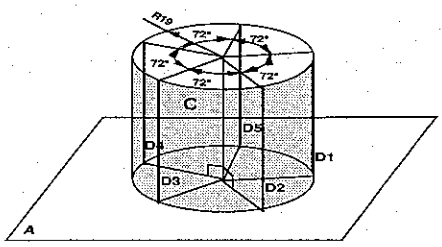

Figure 5

Model :
Elements :
- *5 straight lines Di*
- *a cylinder C*
Internal constraints :
- *the diameter of the cylinder C is 38mm*
- *the lines Di are at regular intervals on C*
Constraints on the reference elements :
- *the cylinder C is perpendicular to the plane A*
Function to be minimised :
- *the maximum of the distances between the centres of the section to the corresponding lines Di*

4.2.General formulation

Taking these three concepts it can be seen that the construction of a defect is expressed in the form of an constrained optimisation. In order to illustrate this the following formulation, which is closer to the structure of the optimisation problems generally encountered, is proposed :

To optimise [function] while respecting
[constraint 1]
...
[constraint i]
...
[constraint n]

In the case of a verification it is sufficient to search for the existence of a realisable solution for the same type of problems. Therefore the simplified formulation is the following :

Check that the following constraints can be respected :
[constraint 1]
...
[constraint i]
...
[constraint n]

In accordance with the preceeding concepts the constraints [constraint i] could be one of two types :

- *internal constraints and constraints on the datum elements, i.e. constraints between the theorical elements :*
  cylinder perpendicular to a plane
  straight lines at regular intervals on a cylinder
  coaxial cylinders
  ...
- *constraints due to the relative situation of the points measured with respect to the model :*
  points interior of a cylinder
  points contained between two planes
  ...

The objective function [function] may be one of the following types :

distance between two planes
diameter of a cylinder
the largest distance between the set of points and the model
the sum of the squares of the distances between a set of points and the model (the standard is not respected).
...

4.3.Resolution

To solve the problem the constraints and the objective function being considered must be translated into mathematical expressions. The model is defined by the parameters and variables of the problem.

Therefore, an objective function is obtained which is to be minimised under certain constraints. This translation must be simplified as much as possible because the expressions are non-linear functions of the variables, a linearisation is indispensable if a quick solution is required. The principal tool used in the linearisation of the expressions is the small-displacement screw [2].

The resolution tools which were used in the cases previously encountered were the following :

- Gauss's method
- simplex method
- Nelder-Mead simplex method

5.EXPERIMENTAL RESULTS

5.1.Experimentation

The results presented are relative to one realized part described in figure 3. This part was measured at 8 points on the surface A, 16 points on the surface B and 12 points for each hole. For each of these holes, the 12 points are situated on 4 sections, and 4 section centres are determined (the centre of the circle passing through 3 points).

The processing was carried out with the help of an experimental computer model which accepts the input of the problem in the form of constraints and functions to be optimised.

5.2.Results

First of all the results obtained for the plane A are given. A2 is the plane obtained by the least-squares method and At the plane obtained by Chebyshev's criterion.

	A2	At
Range of the interval	$2{,}7.10^{-3}$ mm	$2{,}3.10^{-3}$ mm
Angle between A2 and At	$1{,}8.10^{-5}$ rad	

Figure 6

The different results obtained for the localization are illustrated in the table below. The general model is composed of 5 straight lines Di at regular intervals on the cylinder C, with in certain cases supplementary constraints explained in the table. B2 is the straight line prependicular to A2 and passing trough the intersection of the associated cylinder by the least-squares of the surface B with the plane A2. Bt is the cylinder perpendicular to At with a minimum diameter including all the points measured on B.

The optimisation criterion is either the least-squares criterion (L.S.) or Chebyshev's criterion.

Spécification	Contraints	Criterion	Résult
⌖ ⌀ 0,2 A B	C coaxial to B2 D1 passing through a centre		0,207
	C coaxial to B2	L.S.	0,173
	C coaxial to Bt	Chebyshev	0,169
⌖ ⌀ 0,15 A	C coaxial to B2 D1 passing through a centre		0,207
	C perpendicular to A2	L.S.	0,098
	C perpendicular to At	Chebyshev	0,086
⌖ ⌀ 0,1	C coaxial to B2 D1 passing through a centre		0,207
		L.S.	0,089
		Chebyshev	0,071

Figure 7

5.3.Analysis

As a result of what has been previously described on the comparison of the calculation methods, the results of specification which has a datum can only be classified in as much as the datum are correct. Also it should be pointed out that for these cases the results obtained by the different formulations of the problem vary significantly.

On the other hand these datum can be compared for, at least, the plane. By considering the result given for the flatness, according to Chebychev's criterion, as the true value, an error of 18% is obtained. For the localization, that which is more interesting is the angular position of A2 relative to At. In order to estimate the error due to this angle, it is multiplied by the length of the toleranced cylinders ($5x10^{-4}$mm), a value which is negligible when compared with the localization defects. This comes from the fact that the form defect of A is negligible when compared with these defects. To show the influence which the datum model can have on the result, when the form defect of the datum elements and the specified defect are of the same order of magnitude and when the sizes of these elements are also of the same order of magnitude, the angle is multiplied by the largest dimension of the datum element (for example the diameter of the cylinder B limiting the surface A) and $9.5x10^{-4}$mm is obtained. This value is compared to the flatness defect and in this example gives a result of 41%, a non-negligible percentage.

In the case without any datum, a large disparity in the results was observed. Considering that the "true value" of the defect was successfully obtained, the error for the other formulations can be calculated. Therefore, the utilisation of the least-squares criterion on the same model gives an error of 25%. As regards the method which uses a complete datum (non-specified, but compulsory "to find" a value with most software packages), a 191% error was obtained.

6.CONCLUSIONS

The study proposed in this paper shows that all standardized specification verifications by measurement becomes a constrained optimisation problem.

The results obtained illustrate the importance of these an in-depth analysis of the specifications and it's formulation.

The proposed solution methods rigorously solve the standardization problems, however they do present the inconvenience of being slower than those used in actual software packages. It is clear that this inconvenience will lose it's importance as the methods of computation evolve.

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