A Study of Optimal-Criteria Identification Based on the Small-Displacement Screw Model

P. Bourdet (2), LUPRA-ENS Cachan / France; A. Clément (1), ISMCM / France
Received on January 18, 1988

SUMMARY :

The geometric identification of a N-point measured surface may be obtained by means of 4 different optimalization criteria : least-squares on the error form, minimum form error, greatest interior tangential surface, smallest exterior tangential surface.

In this paper a general and unique identification model based on the small-displacement screw is shown to be usefull for any surface and any criterion.

On the basis of experimental results, dealing with planes, circles and cylinders, we have established a comparaison between the different optimalizing criteria.

I -A GEOMETRICAL AND GENERAL IDENTIFICATION METHOD USABLE WITH ANY SURFACE AND ANY CRITERION

Measuring a mechanical part consists of, first, determining the position of a model-surface, and, second, to determining the parameter values of this model surface. Both steps are based on experimental measurements of some points.

Solving this kind of problem is widely called "solving an inverse problem" (TAR. 87). Because of its simplicity, the least-squares criterion (L_2-norm criterion) is widely used for the resolution of inverse problems.

The least-squares criterion is intimately related to the hypothesis of Gaussian uncertainties. For other types of uncertainties, better criteria exist. Among them those based on an L_1-norm (least absolute values) or L_∞-norm (minimax) have the advantage of allowing an easy geometrical formulation.

I.1 - Principle :

We will use here the small displacement screw method already defined (BOU.76) by using L_2 -norm criteria, extending it to three new criteria of geometrical optimization (least form error : L_∞-norm criterion or minimax ; greatest interior tangential surface and smallest exterior tangential surface : L_1-norm criterion).

The manufacturing surface is supposed to be known under the most general way : that is to say by a set of theoretical points M_{ith}, with their normals $\vec{n_i}$,related with the measured error $\xi_i = \overrightarrow{M_{ith}M_i} \cdot \vec{n_i}$ between the measured point and the theoretically defined point.(see Figure 1)

I.2 - Setting up the equation by using small displacements screw model.

I.2.1 We have demonstrated in the past that the most general displacement may be linearized by using the screw model so that the translation displacement vector at point A may be related to the translation displacement vector at point B. (belonging to the same solid), through a vectorial relationship :

$$\vec{D_A} = \vec{D_B} + \vec{AB} \wedge \vec{R} \qquad (1)$$

The new vector $\vec{R}$ is the rotational vector. It can be used instead of the rotational matrix when approximately $|\vec{R}| \leqslant 5°$. The double vector field $\{\vec{R};\vec{D_A}\}$ is a screw. We have designated it : small displacements screw.

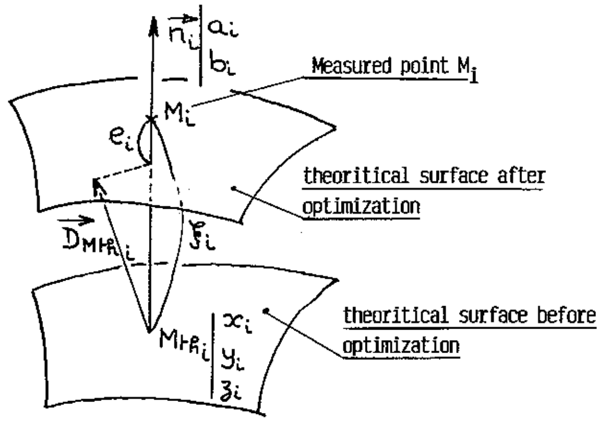


Figure 1

I.2.2 Setting up the equation. We will take as unknown the 6 scalar unknown components of the small displacement screw which, applied to the model-surface, will optimize the residual errors between measured points and theoretically-defined points, according to a L_K-norm criterion. We can write for each measured point :

$$e_i = \xi_i - \left[\vec{D_{Mthi}} \cdot \vec{n_i}\right] \qquad (2)$$

ξ_i represents the error before optimization
e_i represents the error after optimization.

By using the relation (1) we can obtain:

$$\vec{D_{Mthi}} = \vec{D_A} + \overrightarrow{M_{thi}A} \wedge \vec{R} \qquad (3)$$

Then the relation (2) becomes :

$$e_i = \xi_i - (\vec{D_A} \cdot \vec{n_i} + (\overrightarrow{A M_{thi}} \wedge \vec{n_i}) \cdot \vec{R})$$

We can obtain a similar relationship for the p measured point, that is, p linear equations with 6 unknowns (α, β, γ, u, v, w) : the 6 unknown components of the small displacement screw. In the case of mechanical metrology, the nomber p of measured points is always greater than the number r of independent unknowns.

We must therefore determine the optimal value of the small displacement screw according to the criteria which will minimize the distance Z between model and manufactured surface.

If we use the L_1-norm criteria : (least absolute values):

$$Z_1 = \sum_{i=1}^{r} |e_i|$$

If we use the L_2-norm criteria (least-squares)

$$Z_2 = \left[\sum_{i=1}^{r} e_i^2\right]^{1/2}$$

If we use the L_∞-norm criterion : (minimax):

$$Z_\infty = \underset{i}{MAX} |e_i|$$

In each case we must find the small displacement screw so that Z_k must be minimum. It must be emphasized that the L_k-criterion is justified only if the assumption of errors distribued in accordance with a generalized Gaussian probability density f_k (X) of order K is acceptable.

$$K = 1 \quad f_1(X) = \frac{1}{2\sigma_1} \cdot \exp\left[-\frac{|X-X_0|}{\sigma_1}\right]$$

$$K = 2 \quad f_2(X) = \frac{1}{\sqrt{2\pi}\,\sigma_2} \cdot \exp\left[-\frac{1}{2}\frac{(X-X_0)^2}{\sigma_2^2}\right]$$

$$K = \infty \quad f(X) = \begin{cases} \frac{1}{2\sigma_\infty} & \text{for } X_0 - \sigma_\infty \leq X \leq X_0 + \sigma_\infty \\ 0 & \text{otherwise} \end{cases}$$

Strictly speaking, we can use K = 2 (least-squares criterion) only if the errors are distribued following a true Gaussian probability density. In mechanical metrology this case never happens ; nevertheless, everyone uses it. Conversely, the case $K = \infty$(minimax criterion) which always happens on a measuring machine with limited errors is never used. But rather than to try identifying the probability density errors function on the manufactured surfaces, we prefer to give the geometrical meaning of each criterion.

I.3 - First criterion of optimization : least squares on form error :

There is nothing new to say about the L_2 - norm criterion except that it is very sensitive to some aberrations in the data. In our case we have to find the small displacement screw T_A in order that:

$$Z_2^2 = \sum_{i=1}^{r} (e_i)^2 \quad \text{be minimum.}$$

The result can be readily compute by solving a linear system, whatever the surface to be identified (cylinder, cône, complex surface, ...).

I.4 - Second criterion of optimization : minimum error form (According to I.S.O.).:

We will solve the L_∞-norm minimization by using linear programming. We have to find the small displacement screw T_A in order that :(see Figure 2)

$$Z_\infty = \underset{i}{MAX} \left| Sup(e_i) - INF(e_i) \right| \quad \text{be minimum.}$$

That is to say, using the linear constraints,

$$\Delta S - e_i \geq 0 \qquad i \in \{1 - r\}$$
$$\Delta I - e_i \leq 0 \qquad i \in \{1 - r\}$$

the function, $Z_\infty = \Delta S - \Delta I$ will be minimum.

ΔS representes the upper band of e_i's

ΔI representes the lower band of e_i's

Z_∞ representes the error form ISO's definition.

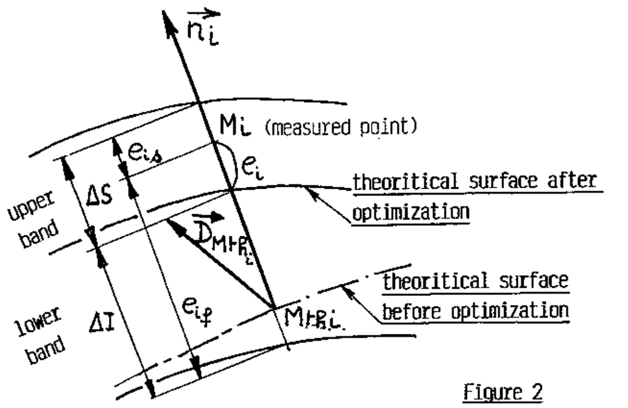


Figure 2

I.5 - Third and fourth criteria of optimization : smallest exterior tangential surface and greatest interior tangential surface :

In the case in which we seek the greatest interior tangential surface, we must choose as a model of the initial surface an interior surface such that the e_i's be positive (thus eliminating the sign problem). We must then find the small-displacement screw T_A in such a manner that under the linear constraints

$$\Delta S - e_i \geq 0$$

the function $Z_1 = \Delta S$ will be minimum.

Similarly, when we seek the smallest exterior tangential surface, we must choose as a model of the initial surface an exterior surface such that the e_i's be positive. We must then find the small-displacement screw T_A in such a way that under the linear contraints.

$$\Delta I - e_i \leq 0$$

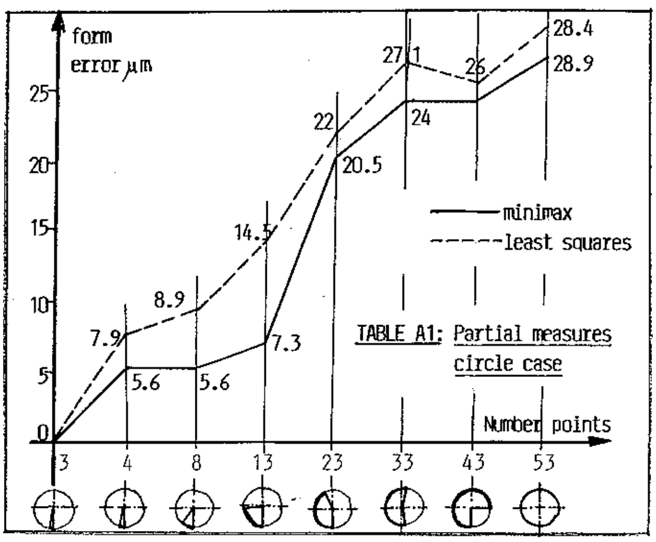
the function $Z_1 = -\Delta I$ will be minimum.

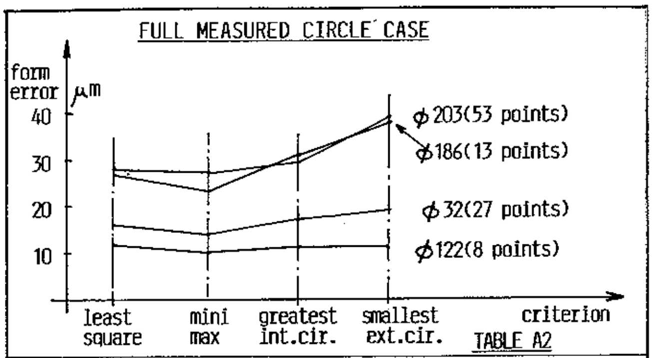
II - COMPARING RESULTS OBTAINED FROM EXPERIMENTATION ON CIRCLES, PLANES AND CYLINDERS :

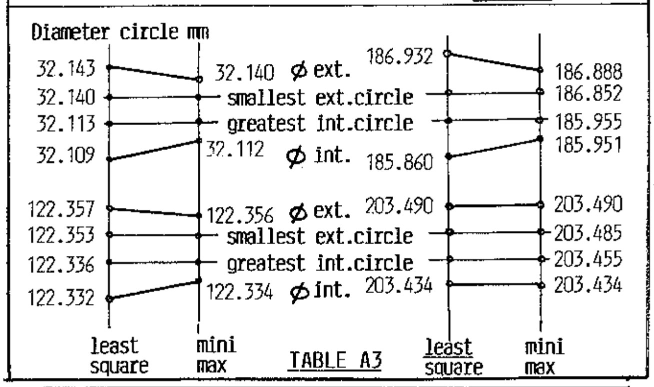
These experimental results were obtained using a measuring machine (S.E.I.V.-RENAULT - PROMESUR SOFTWARE) whose absolute precision within the entire volume is ± 3,5 μm including probe errors.(KUN.83) We measured 3 kinds of surface using the 4 criteria previously defined. We compared the results from 3 differents points of view :(MIR.86)

- Form error,
- Model-surface final position and its related geometrical parameters
- Calculation time.

II.1 - Circles







Pts Nbr	Least-squares	Minimax	Gain	Gain %
4	0.3457	0.3354	0.0103	3 %
16	0.0380	0.0325	0.0055	16 %
8	0.0126	0.0108	0.0118	16 %
27	0.0167	0.0140	0.0027	19 %
13	0.5360	0.4687	0.0673	14 %
53	0.0284	0.0279	0.0005	1.8 %

Circle: Forme error TABLE A4

In all cases, we find that the minimax criterion is an improvement over the least-squares criterion with regard to form errors. This improvement, which is about 15 % in most cases, (table A2-A4) becomes as great as 98 % when the circle is probed only on a part of its circumference less than πD/4. (table A1). We also see that the exterior circle and interior circle diameters obtained from the minimax are always closer to the greatest circle and smallest circle respectively than those obtained from least-squares (table A3). Finally we see that in any case in which the probing zone is greater than πD/4, the minimax criterion and the least-square criterion give a similar spread on diameter and center position. With few probing points, the spread is smaller with minimax than with the least-square. With a probing zone smaller than π D/4, the results are erratic in any case.

N.B. : Particular case : Probing circle in (NAW.81) 4 opposed points on 2 perpendicular diameters. This case is widely used in everyday practice.

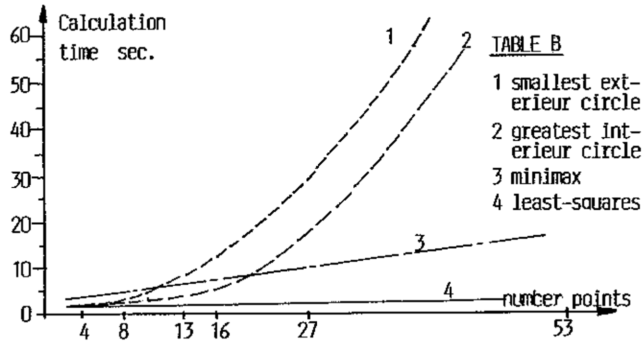
The ellipse passing through these 4 points has the same center as the minimax circle.

The least-squares circle has the same center as the circle obtained by intersection of the 2 mediators of the 2 orthogonal chords.

The greatest interior circle and the smallest exterior circle pass through 3 measured points and their centers are far away from both least-squares and minimax criteria.(according to the well-known properties of L_1-norm criterion).

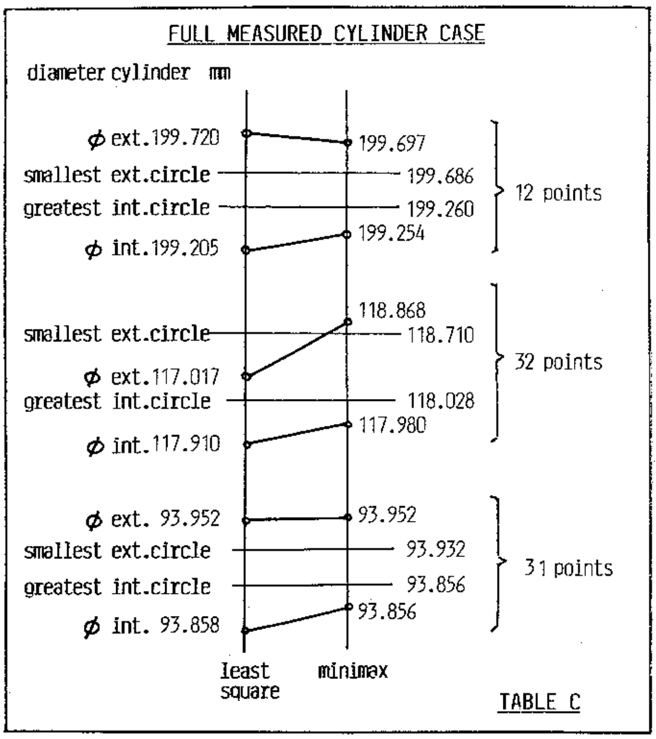
Influence of the number of measured points on calculation time.

The results, shown below, present no surprises (TABLE B). Each calculation time has been measured on a small HP 9826 non-compatible computer.



II.2 - Cylinder :

We can make the same observations as in the case of the circle (TABLE C). For example, in form errors the minimax criterion gave results 10 % better than those obtained by the least- squares criterion while the calculation time was multiplied by 15. But this time is within industrial limits when the number of measured points is few. The greatest interior cylinder and the smallest exterior cylinder (L_1 - norm criterion) can be computed only when the measured points are situated on more than the half-cylinder. As with the circle, the minimum diameter and the maximum diameter obtained by minimax criterion are closer than the greatest cylinder and smallest cylinder obtained by the least-squares criterion.



II.3 - Plane :

Pts Nbr	Least-squares	Minimax	Gain %
6	0.0096	0.0083	15 %
9	0.0116	0.0106	10 %
30	0.0216	0.0201	7 %
53	0.1727	0.1647	5 %

Plane: Forme error TABLE D1

We see that form error increased with the number of measured points. Conventionally a number of points between 4 and 12 is most often used. Improvement in form error is then visible when the minimax criterion is used. By contrast, when the number of points is very large, using the minimax criterion yields no true improvement in form error. Because of the time consumed the least-squares criterion is then better.

III COMPARING THE EFFECTS OF AN ERRATIC POINT :

III.1 CIRCLE :

We have inserted in the data list of measured points on a circle with 16 μm form error, an aberration point in such a manner that the $\xi_i = 9 \mu m$ of a measured point becomes $\xi_i = 210 \mu m$.(TABLE E)

	Least-squares		Minimax	
	No aberrant pt	With aberrant pt	No aberrant pt	With aberrant pt
Form error	0.016	0.225	0.014	0.218
Center X Circle Y	160.230 142.805	160.222 142.290	160.232 142.804	160.184 142.706
ϕCircle	32.126	31.946	32.126	32.122

Circle: aberant point TABLE E

The difference between the results obtained by minimax and those obtained by least-squares is exactly the same, with or without aberration point. So, greater the form error value, the greater the probability of aberration point. The center circle position is more perturbed with the minimax than with the least-squares criterion, but both are perturbed (respectively 109μm and 17μm). The least-squares mean circle diameter is strongly perturbed (+180 μm) but the minimax mean circle diameter is almost unchanged (+ 4 μm). It is evident from these results that it is easy to determine an aberration point with the least-squares criterion : An error e_i greater than $m \pm 3\sigma$ must be like an aberration point with probability of 93,72 %. Conversely it is almost impossible to determine an aberration point with the minimax criterion.

III.2 PLANE :

As in the case of the circle, we have inserted in the data list for a plane with 20 μm of form error, an aberration point in such a manner, that its ξ_i has incrased by 173 μm.(TABLE F)

	Least-squares		Minimax	
	No aberrant Pt	With aberrant Pt	No aberrant Pt	With aberrant Pt
Form error	0.0216	0.1630	0.0201	0.1449
Plane $\vec{n} \begin{cases} a \\ b \end{cases}$	0.002757 0.000203	0.003085 -0.000062	0.002798 0.000274	0.004033 -0.000528

Plane: aberrant point TABLE F

As in the previous case, the value of the form error indicates the possibility of the aberration point, and only the least-squares criterion makes it possible to perform the statistical analysis for eliminating it.

IV - GENERAL RESULTS ANALYSIS :

- The minimax criterion improved the form error obtained with the least-squares criterion by about 15 %. This improvement becomes almost null when the number of points is greater than 20.
- In any case the L_1-norm criterion (smallest or greatest tangential surface) is closer to the minimax criterion than to the least-squares criterion.
- The distributing measurements points over the whole surface gives the greatest effect.
- In the case of circle and cylinder probing on a part of its circumference lesser than $\pi D/4$, neither the minimax norm the least-squares criterion gives a good result.
- With a very great number of measured points, only the least-squares, criterion makes it possible to detect the aberration points, but in any case the value of the form error indicates the aberration point.

We can thus state that with a very great number of measured points, the least-squares criterion provides :

- Time-saving.
- Statistical analysis of the precision and the aberration points.
- Form error almost equal to the minimax value.

By contrast, when the number of measured points is fewer than 12 (as in everyday practice), the minimax criterion must always be chosen because it gives a result conforms with ISO-norm in a reasonable calculation time. Unlike the criterion of least-squares, the criteria of greatest internal surface, least external surface and minimax conform to the specifications of the ISO-norm.

REFERENCES

(BOU. 76) P.BOURDET, A. CLEMENT,
"Controlling a complex surface wxith a 3 axis Measuring machine."
Annals of C.I.R.P., vol. 25/1/1976 p. 354-361.

(KUN. 83) H. KUNZMANN, F. WALDELE,
"On Testing Coordinate Measuring Machines (CMM) with Kinematic Reference Standards (KRS),
Annals of the C.I.R.P., vol. 32/1/1983.

(MIR. 86) Th. MIRONNEAU,
"Etude comparative des méthodes d'optimisation des surfaces en métrologie dimensionnelle."
LURPA ENS CACHAN - Rapport interne Juin 1987.

(NAW. 81) NAWARA, M. KOWALSKI, J. KACZMARCK,
"Investigation on the variability of the result of measurements realized by multicoordinate measuring machines on circular section with determined types of roundness error."
Annals of the C.I.R.P., vol. 30/1/81, p. 437.

(TAR.87) A.TARANTOLA
"Inverse problem theory". Elsevier Edition.