UNCERTAINTY EVALUATION FOR POSE ESTIMATION BY MULTIPLE CAMERA MEASUREMENT SYSTEM

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In this paper, a multiple camera measurement system is presented for the pose (position and orientation) estimation of a robot end-effector for assembly. First, a multiple camera geometric model is introduced for the 3D pose estimation of the end-effector and the identification of the intrinsic and extrinsic parameters of cameras. Second, an uncertainty propagation model is derived from the geometric model that enables the number of cameras to be chosen, the relative position between cameras and finally uncertainty evaluation for pose estimation. Experimental results are presented in the last section of this paper.

1. Introduction

Modern industrial trends in aeronautics and space have placed an increasing demand on the subject of robotics and robot manipulators and hence the attention of many researchers from different engineering and science areas. The emphasis on sensor-guided robots, off-line programming, robot calibration, to name a few, contributed to reach some sort of maturity and important research results toward efficiency and ease of use of robots for many applications (assembly and welding, product inspection and testing,. . .) through the enhancement of the accuracy, the speed and the flexibility of robots.

Robot calibration is the process of enhancing the accuracy of a robot manipulator through modification of the robot control software [1]. Calibration process encompasses four steps: Kinematic modeling, Pose measurement, Kinematic identification and Kinematic compensation. A wide account of robot calibration literature is provided in [2], [3] and [4].

Robot calibration can be done using multiple cameras setup. Position measurement using cameras first requires careful calibration of the cameras. These calibrated cameras define afterward the robot world coordinate frame. Optimal camera alignment [5] (number of cameras, relative pose) and evaluation of the accuracy of the resulting system is an important issue toward pose measurement.

In this paper, a multiple camera measurement system is presented for the pose estimation of a robot end-effector for assembly applications requiring extremely tight tolerances. First, a multiple camera geometric model is introduced for the 3D pose estimation of the end-effector and the identification of the intrinsic and extrinsic parameters of cameras. Second, an uncertainty propagation model is derived from the geometric model that enables the number of cameras to be chosen, the relative position between cameras and finally uncertainty evaluation for pose estimation. Experimental results and software development are presented in the last section of this paper.

2. Multiple camera geometric model

2.1. *The Pin-hole model*

The Pin-hole model [6] assumes a correspondence between real object point and image point through a straight line that passes through the focal point of the camera. Its also described as the linear projective transformation from the projective space $\Re^3$ into the projective plane $\Re^2$ represented as follows:

$$\mathrm{M_{Cam}} = \begin{pmatrix} \mathrm{fx} & 0 & \mathrm{u_0} & 0 \\ 0 & \mathrm{fy} & \mathrm{v_0} & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \mathrm{r_{11}} & \mathrm{r_{12}} & \mathrm{r_{13}} & \mathrm{t_x} \\ \mathrm{r_{21}} & \mathrm{r_{22}} & \mathrm{r_{23}} & \mathrm{t_y} \\ \mathrm{r_{31}} & \mathrm{r_{32}} & \mathrm{r_{33}} & \mathrm{t_z} \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \mathrm{m_{11}} & \mathrm{m_{12}} & \mathrm{m_{13}} & \mathrm{m_{14}} \\ \mathrm{m_{21}} & \mathrm{m_{22}} & \mathrm{m_{23}} & \mathrm{m_{24}} \\ \mathrm{m_{31}} & \mathrm{m_{32}} & \mathrm{m_{33}} & \mathrm{m_{34}} \end{pmatrix} \tag{1}$$

The camera matrix $\mathrm{M_{Cam}}$ contains intrinsic parameters (the scale factors and the image center) and extrinsic parameters (the transformation from world coordinates to the camera coordinates).

The world coordinates and the image coordinate systems are related by the following equations under undistorted camera model assumptions:

$$\begin{cases} \mathrm{u_C} = \dfrac{\mathrm{m_{11}X_C + m_{12}Y_C + m_{13}Z_C + m_{14}}}{\mathrm{m_{31}X_C + m_{32}Y_C + m_{33}Z_C + m_{34}}} \\ \mathrm{u_C} = \dfrac{\mathrm{m_{21}X_C + m_{22}Y_C + m_{23}Z_C + m_{24}}}{\mathrm{m_{31}X_C + m_{32}Y_C + m_{33}Z_C + m_{34}}} \end{cases} \tag{2}$$

It is now possible to express 3D world coordinates from the camera coordinates $(\mathrm{u}_{i\mathrm{C}}\ \mathrm{v}_{i\mathrm{C}})^{\mathrm{T}}$ when considering multiple camera system $(1 \leq i \leq n)$ by solving a standard least square fitting which minimize the location error of points in the image plane e = A× ***b*** - c described as follows:

$$
\mathrm{A} = \begin{pmatrix} & \vdots & \\ \mathrm{m}_{11,i} - \mathrm{m}_{31,i}\mathrm{u}_{i\mathrm{C}} & \mathrm{m}_{12,i} - \mathrm{m}_{32,i}\mathrm{u}_{i\mathrm{C}} & \mathrm{m}_{13,i} - \mathrm{m}_{33,i}\mathrm{u}_{i\mathrm{C}} \\ \mathrm{m}_{21,i} - \mathrm{m}_{31,i}\mathrm{v}_{i\mathrm{C}} & \mathrm{m}_{22,i} - \mathrm{m}_{32,i}\mathrm{v}_{i\mathrm{C}} & \mathrm{m}_{23,i} - \mathrm{m}_{33,i}\mathrm{v}_{i\mathrm{C}} \\ & \vdots & \end{pmatrix}
$$
$$
\mathrm{c} = \begin{pmatrix} \vdots \\ \mathrm{m}_{34,i}\mathrm{u}_{i\mathrm{C}} - \mathrm{m}_{14,i} \\ \mathrm{m}_{34,i}\mathrm{v}_{i\mathrm{C}} - \mathrm{m}_{24,i} \\ \vdots \end{pmatrix} \quad \boldsymbol{b} = \begin{pmatrix} \mathrm{X_C} \\ \mathrm{Y_C} \\ \mathrm{Z_C} \end{pmatrix} \tag{3}
$$

The result can be written:

$$
\begin{cases} \mathrm{X_C} = \mathrm{f}_1(\ldots, \mathrm{u}_{i\mathrm{C}}, \mathrm{v}_{i\mathrm{C}}, \ldots) \\ \mathrm{Y_C} = \mathrm{f}_2(\ldots, \mathrm{u}_{i\mathrm{C}}, \mathrm{v}_{i\mathrm{C}}, \ldots) \\ \mathrm{Z_C} = \mathrm{f}_3(\ldots, \mathrm{u}_{i\mathrm{C}}, \mathrm{v}_{i\mathrm{C}}, \ldots) \end{cases} \tag{4}
$$

2.2. *Pose calculation*

Least square fitting is also used to calculate the pose of a marked object. The position of markers on the object $\boldsymbol{C}_{\mathbf{Obj},j}$ $(\mathrm{X}_{\mathrm{Obj},j}\ \mathrm{Y}_{\mathrm{Obj},j}\ \mathrm{Z}_{\mathrm{Obj},j})^{\mathrm{T}}$, $1 \leq j \leq p$ (p markers) is known and $\boldsymbol{C}_{\mathbf{Meas},j}$ $(\mathrm{X}_{\mathrm{Meas},j}\ \mathrm{Y}_{\mathrm{Meas},j}\ \mathrm{Z}_{\mathrm{Meas},j})^{\mathrm{T}}$, $1 \leq j \leq p$ is measured by the multiple camera system yielding to the following equation where $\mathrm{P}_{\mathrm{Obj,Meas}}$ is the Pose Matrix:

$$
\begin{aligned} &\boldsymbol{C}_{\mathbf{Meas},j} = \mathrm{P}_{\mathrm{Obj,Meas}} \cdot \boldsymbol{C}_{\mathbf{Obj},j} \\ &\text{or} \\ &\mathrm{P}_{\mathrm{Obj,Meas}} = \mathrm{g}(\boldsymbol{C}_{\mathbf{Meas},j}, \boldsymbol{C}_{\mathbf{Obj},j}), 1 \leq j \leq p \end{aligned} \tag{5}
$$

2.3. *Visibility model*

To track the robot end-effector, all its positions in the working volume need to be visible through the placed reference markers on the end-effector.A Marker Visibility Volume (a set of geometrical properties that allow an optimal camera position) is defined for each marker position on the robot end-effector.

Visibility volumes intersection and camera field of view enables then to extract a set of optimal positions and orientations of cameras (Fig. 1).

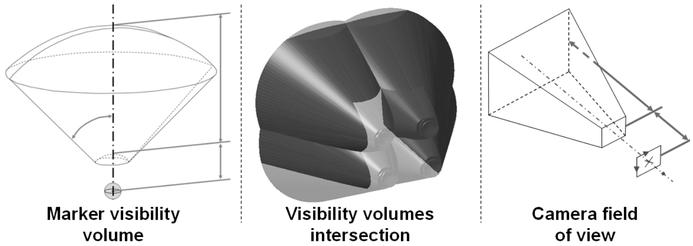

Fig. 1. Visibility model for the optimal relative pose of cameras

3. Uncertainty evaluation

From a metrological view camera-based measurement have a stated measurement uncertainty. In real applications the final measurement uncertainty is affected by many factors such as illumination, edge effects, the operator, and non-regularities of the object to be measured.

The study carried out here explores uncertainty on the position of the centers of markers when their position on the camera coordinate system is measured (Eq. 3 and Eq. 4).

According to [7] and [8] several approaches to propagating uncertainty could be used. One approach is to use Monte Carlo methods, which is highly general but computationally expensive. Our approach uses an analytical method based on covariance propagation method expressed as follows:

$$\mathrm{Cov}(\mathrm{f}\ (\ldots, \mathrm{u}_{i\mathrm{C}}, \mathrm{v}_{i\mathrm{C}}, \ldots)) = \mathrm{J}_{\mathrm{f}/\vec{\boldsymbol{u}}} \times \mathrm{Cov}(\vec{\boldsymbol{u}}) \times \mathrm{J}^{\mathrm{T}}_{\mathrm{f}/\vec{\boldsymbol{u}}}, 1 \le i \le n$$

$$\mathrm{Cov}(\vec{\boldsymbol{u}}) = \begin{pmatrix} \ddots & & & 0 \\ & \mathrm{Var}(\mathrm{u}_{i\mathrm{C}}) & \mathrm{cov}(\mathrm{u}_{i\mathrm{C}},\mathrm{v}_{i\mathrm{C}}) & \\ & \mathrm{cov}(\mathrm{v}_{i\mathrm{C}},\mathrm{v}_{i\mathrm{C}}) & \mathrm{Var}(\mathrm{u}_{i\mathrm{C}}) & \\ 0 & & & \ddots \end{pmatrix} \quad (6)$$

$$\mathrm{J}_{\mathrm{f}/\vec{\boldsymbol{u}}} = \begin{pmatrix} \frac{\partial \mathrm{f}_1}{\partial \mathrm{u}_{1\mathrm{C}}} & \frac{\partial \mathrm{f}_1}{\partial \mathrm{v}_{1\mathrm{C}}} & \cdots & \frac{\partial \mathrm{f}_1}{\partial \mathrm{u}_{n\mathrm{C}}} & \frac{\partial \mathrm{f}_1}{\partial \mathrm{v}_{n\mathrm{C}}} \\ \frac{\partial \mathrm{f}_2}{\partial \mathrm{u}_{1\mathrm{C}}} & \frac{\partial \mathrm{f}_2}{\partial \mathrm{v}_{1\mathrm{C}}} & \cdots & \frac{\partial \mathrm{f}_2}{\partial \mathrm{u}_{n\mathrm{C}}} & \frac{\partial \mathrm{f}_2}{\partial \mathrm{v}_{n\mathrm{C}}} \\ \frac{\partial \mathrm{f}_3}{\partial \mathrm{u}_{1\mathrm{C}}} & \frac{\partial \mathrm{f}_3}{\partial \mathrm{v}_{1\mathrm{C}}} & \cdots & \frac{\partial \mathrm{f}_3}{\partial \mathrm{u}_{n\mathrm{C}}} & \frac{\partial \mathrm{f}_3}{\partial \mathrm{v}_{n\mathrm{C}}} \end{pmatrix}$$

$\mathrm{Cov}(\vec{\boldsymbol{u}})$: Covariance matrix of centers of a marker on n image planes
$\mathrm{J}_{\mathrm{f}/\vec{\boldsymbol{u}}}$: Jacobian matrix of f around $\boldsymbol{C}$ $(1 \le i \le n)$ on each camera

$$\mathrm{Cov}(\mathrm{f}(\vec{\boldsymbol{u}})) = \begin{pmatrix} \mathrm{Var}(\mathrm{X_C}) & \mathrm{cov}(\mathrm{X_C}, \mathrm{Y_C}) & \mathrm{cov}(\mathrm{X_C}, \mathrm{Z_C}) \\ \mathrm{cov}(\mathrm{Y_C}, \mathrm{X_C}) & \mathrm{Var}(\mathrm{Y_C}) & \mathrm{cov}(\mathrm{Y_C}, \mathrm{Z_C}) \\ \mathrm{cov}(\mathrm{Z_C}, \mathrm{X_C}) & \mathrm{cov}(\mathrm{Z_C}, \mathrm{Y_C}) & \mathrm{Var}(\mathrm{Z_C}) \end{pmatrix} \quad (7)$$

Once we obtain the covariance matrix for a given marker in the measurement space, we can apply the same method to estimate uncertainties for p points of an object seen by n cameras to extract uncertainties on the orientation and the position of the object in the measurement space:

$$\mathrm{Cov}(\mathrm{g}(\boldsymbol{C}_{\mathbf{Meas},j}, \boldsymbol{C}_{\mathbf{Obj},j})) = \mathrm{J}_{\mathrm{g}/\vec{\mathbf{c}}} \cdot \mathrm{Cov}(\boldsymbol{C}_{\mathbf{Meas},j}, \boldsymbol{C}_{\mathbf{Obj},j}) \cdot \mathrm{J}^{\mathrm{T}}_{\mathrm{g}/\vec{\mathbf{c}}} \tag{8}$$

4. Experimental results

Experiments based on a two cameras measurement system are shown to validate the uncertainty propagation application. (Fig. 2) depicts the measurement of the center of a marker on each camera (10 000 measurement). (Fig. 3) shows the resulting calculation of the corresponding 3D point (world coordinates system).

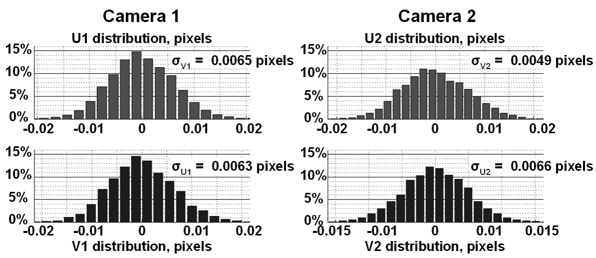

Fig. 2. Experimental results on image planes (2 cameras)

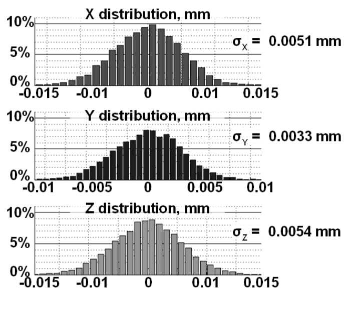

Fig. 3. Experimental results of the corresponding 3D point

Monte Carlo simulation is used here as a sampling method for analyzing uncertainty propagation. Inputs (coordinates on image plane) are randomly generated from probability distributions (normal or uniform) using MINITAB®. Results of the simulation are showed here (Fig. 4 and Fig. 5).

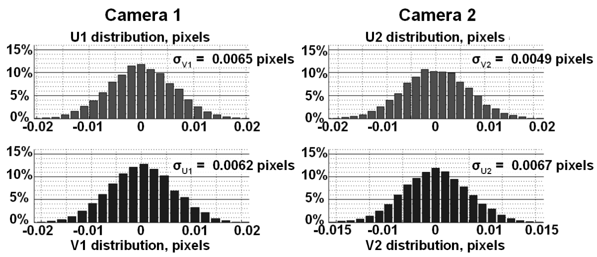

Fig. 4. Sampling on image planes (2 cameras)

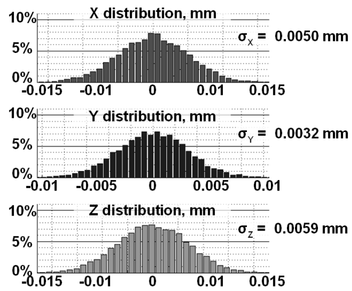

Fig. 5. Stochastic uncertainty propagation results

MAPLE® software has been used to calculate an explicit solution of the analytical model based on covariance propagation (Eq. 6). The covariance matrix of the corresponding 3D point (Eq. 7) is computed and the results as highlighted hereafter show similar results (experimental, stochastic uncertainty propagation and explicit solution for covariance propagation).

$$\mathrm{Cov}(\mathrm{f}(\overrightarrow{\boldsymbol{u}})) = \begin{pmatrix} 2.533e^{-5} & -1.828e^{-6} & -1.828e^{-6} \\ -1.828e^{-6} & 2.533e^{-5} & -1.828e^{-6} \\ -1.558e^{-5} & -1.828e^{-6} & 2.533e^{-5} \end{pmatrix} \begin{matrix} \sigma_\mathrm{X} = 0.0050\mathrm{mm} \\ \sigma_\mathrm{Y} = 0.0032\mathrm{mm} \\ \sigma_\mathrm{Z} = 0.0068\mathrm{mm} \end{matrix} \quad (9)$$

5. Conclusion

This paper presents models used for the setup and the validation of a multiple camera based measurement system for the pose estimation of a robot end-effector.

The multiple camera model based on linear projective transformation on image plane coupled with the visibility model lead to a geometric model for multiple camera measurement (number of cameras and relative position and orientation between cameras) and enable world coordinates calculation and 3D object pose estimation.

The study carried out here explores uncertainty on the position of the centers of markers when their position on the camera coordinate system is measured. The uncertainty propagation approach described here is based on stochastic uncertainty propagation as well as an explicit solution for covariance propagation.

Experimental results showed similar results compared to stochastic uncertainty propagation and explicit solution for covariance propagation when using two cameras.

References

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