TAKING THE DEFORMATION INTO ACCOUNT FOR COMPONENTS' TOLERANCING

Grégory Cid

LURPA, ENS de Cachan, 61 av. du Pdt Wilson, 94235 Cachan cedex, France

Gregory.Cid@lurpa.ens-cachan.fr

François Thiébaut

LURPA, IUT de Cachan, 9, av de la Division Leclerc, 94235 Cachan cedex, France

Francois.Thiebaut@lurpa.ens-cachan.fr

Pierre Bourdet

LURPA, ENS de Cachan, 61 av. du Pdt Wilson, 94235 Cachan cedex, France

Pierre.Bourdet@lurpa.ens-cachan.fr

Abstract:

This paper deals with the tolerancing of assemblies composed of compliant components.

Work in geometrical tolerancing is based on the assumption of rigid components. This model uses mechanism theory, setting in relative position of the components with geometric deviations from the nominal geometry, and giving geometrical equations relating to the system. The synthesis of both specifications and tolerances is carried out to provide the geometrical tolerancing of components of the system.

Work relating to the mechanical behaviour of the systems is based mainly on digital models resulting from the CAD. They make possible to know, according to the boundary conditions in displacements and/or forces, the values of the stress, strain and displacements at the various points of the studied structure.

Currently, the two studies are separated and give independent results. The influence of tolerancing on boundary conditions is not always negligible. The work presented here takes into account both the variations of geometry coming from tolerancing, and the displacements coming from the deformations of the components to provide the global geometric relations of the assembly.

Key words: tolerancing, deformation, assembly

1 Introduction

This paper deals with the tolerancing of assemblies composed of compliant components. The global objective of tolerancing consists in determining the allowed geometry of its component, that means the allowed forms, dimensions and the authorised deviations are expressed through geometrical specifications according to standards.

When the manufactured components are within the specified tolerances, the functional requirements on the assembly are respected.

Problems relative to tolerancing are the following:

- finding geometric specifications,
- finding values of the tolerances (authorised deviations),
- simulating the behaviour of the assembly composed of non ideal components.

In previous work, it has been shown how to simulate the behaviour of assembly composed

of rigid components. This work leads to:
- establish the geometric relations on the assembly that include geometric deviations of components
- define the allowed geometry of the components
- impose the behaviour of interfaces between the components
- simulate the behaviour of the assembly

This first approach is completed in the present paper by integrating the compliance of the components.
Initially it is shown how to obtain the geometric relations linked to the assembly. The integration of the compliance is then presented and an industrial example of assembly is finally proposed to illustrate its application.

2   Geometric behaviour of an assembly composed of rigid components

Throughout the paper, a simplistic assembly is used to illustrate our method of resolution. The planar assembly consists of two components. The first component (rigid body) is called B, and C the second one. Frame associated to B, (respectely to C) is noted R_B (respectely R_C).

The component C is linked to the body B through four rigid and one elastic links. These five links are called L_i, $i \in [1, 5]$. The nominal geometry of the components is supposed to be known. A Body Nominal point, respectively a Component Nominal point, is noted $N(B_i)$, respectively $N(C_i)$..
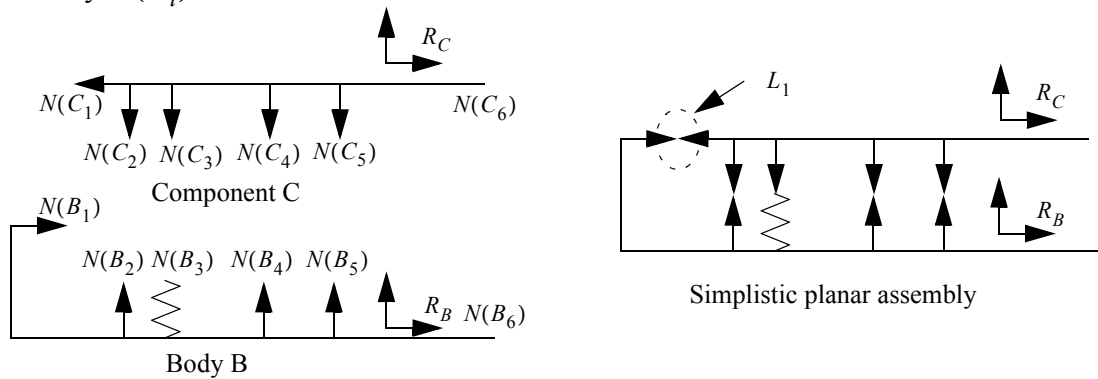

Figure 1: Nominal components of the illustrative assembly

2.1   Geometric relations

There are several ways of representing components of a system with defects by integrating variations of geometry, and several ways of modeling the links between components. In the approach developed by Ballot [1], and Thiébaut [10], the whole set of faces in ideal position of the considered component constitutes the initial geometry of the component. The initial situations of component set of faces are defined relative to a frame associated with that component.
For each component of a mechanical system, manufactured geometry of component is not identical to the initial geometry. The assumption that defects of form are negligible compared to defects of position and orientation of the faces is taken. Thus, each manufactured face has the same type as the initial one. Variations of position and orientation of the situation elements of manufactured faces compared to initial faces represent a model of 3D geometry variations. Since the geometric variations are small, rotation are approximated using a linear model. The small displacement torsor [2] is used to model rotations. Elementary links between two manufactured faces of components are thus described by a 3D model of variations which allows taking into account both their relative positions and orientations. One can find more information on the 3D

variation modeling in [9].

All these 3D models of variations make possible the modeling of the intern geometrical relations of the studied system.

The variations of geometry of the components and the links between the components of the illustrative assembly are very simple.
A model of the manufactured assembly is established. The points of the manufactured component are not identical to the nominal points due to the manufacturing process. The manufactured point positions of the body and the component (figure 2) are named $M(B_i)$ and $M(C_i)$.
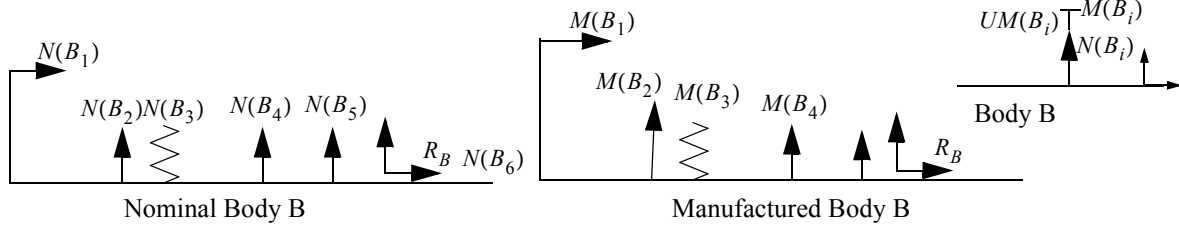

Figure 2: Manufactured component

$M(B_i)M(C_i)$ represents the gap inside of i link. Basically, our approach relies on expressing the symbolic equation corresponding to the link (figure 3). Each link of the assembly provides a similar equation. The whole set of equations constitutes the geometric relations associated to the assembly.
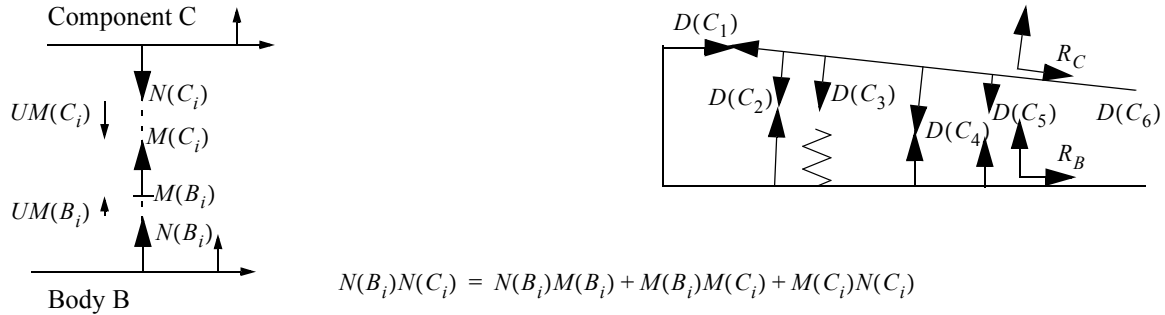

$$N(B_i)N(C_i) = N(B_i)M(B_i) + M(B_i)M(C_i) + M(C_i)N(C_i)$$

Figure 3: Geometric equation of a link

$N(B_i)M(B_i)$ represents the variation of the i link position, it is called $UM(B_i)$.

Since the situation of all nominal points is known in the component frame and all the deviations $N(B_i)M(B_i)$ are fixed for a given component, the only unknowns of the system of equations are the gaps between the components. If it is not possible to find the value of each gap independently of the others, the assembly is geometrically over constrained.

Concerning the illustrative 2D assembly, it is possible to ensure the contact between the body and the component for only three links. figure 3 presents the case of contacts at links 1,2 and 4. One can note that the component frame R_C has deviated from its initial situation relative to the body frame R_B . The points of the manufactured element have moved in the body frame. The variations of point situations between the nominal position and that for which contacts are ensured is named $UD(C_i)$. Contact at link 5 is not possible: the illustrative assembly is geometrically over constrained.

2.2   Allowed geometry

To raise over constraints, it is necessary to impose geometrical conditions which are expressed as geometrical specifications (parallelism...). Concerning the illustrative example, contacts are nearly ensured for every links if the points $M(B_2)$, $M(B_4)$, $M(B_5)$ of the body are nearly aligned within a small tolerance and the points $M(C_2)$, $M(C_4)$, $M(C_5)$ of the component are nearly aligned within a small tolerance.

Geometric relations of the mechanism are obtained in the previous section. It is necessary to limit faces situation variations $(N(B_i)M(B_i))$ in order to simulate the behaviour of the assembly. This operation is carried out by imposing geometrical constraints between the points of the faces using virtual gauges [4]. These geometrical constraints reflect through inequations geometrical specifications. The approach was developed using symbolic calculation. Obtained results from our example show the relevance of the approach for the three-dimensional tolerance of mechanical systems made up of rigid components.

2.3   Behaviour of links

Specificity of the links is taken into account by specifying contacts between components [9]. The contact is said floating, sliding or fixed. A representation of the three types of contacts is given in figure 4.
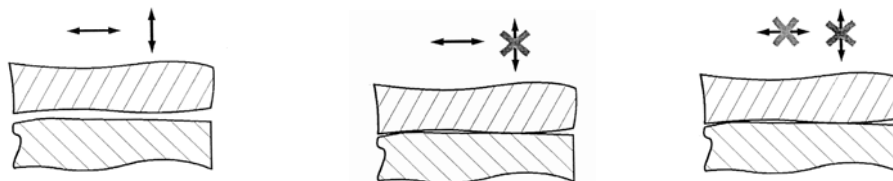

Figure 4: Floating, sliding and fixed contacts

At this stage, the geometric relations that permit the simulation of the assembly are expressed, the allowable geometry is defined and the aims of the designer are imposed by specifying the contact conditions on the discrete geometry.

2.4   Simulation of the behaviour of the assembly

The equations corresponding to the geometric relations and the definition of the admissible geometrical deviations are established. We have all the data that permit to perform a tolerance analysis i.e. to check that functional requirements are respected for each allowed geometry.

The functional requirements are defined through distance functions. The method consists in searching extrema of the distance functions using linear programming.

A set of functions defined [9] that permit to find out:

- the minimum distance between two surfaces for a set of points
- the maximum distance between two surfaces for a given point
- the maximum/minimum angle through a given direction between two geometrical features
- the maximum linear or angular displacement through a given direction between two components

The last step of the worst case analysis method consists in optimizing distance functions as-

sociated with the functional requirements.

The geometrical constraints are the aggregation of the geometric equations, of the contact constraints and of the constraints relative to the allowed geometric deviations.

3   Calculation of the displacement due to the compliance of the components

The mechanical study of the assembly makes it possible to predict the mechanical requests within the links between the components via a dynamic or static study. In addition to this study undertaken on no compliant components, the study of the constraints and the deformations of the components is carried out. In a first approach, the displacement is supposed to be linear relative to the force. The rigidity which binds the forces F to the displacement UC is established through finite element modeling in the form of a matrix (equation 1).

$$[F] = [K][UC] \tag{1}$$

Considering our example:

- the points $M(C_1)$, $M(C_2)$, $M(C_4)$ define the situation of the component relative to the body, null displacements are imposed on these points
  $(UC(C_1) = 0, UC(C_2) = 0, UC(C_4) = 0)$
- the point $M(C_5)$ can not be in contact with the point $M(B_5)$, a condition of non null displacement is imposed: $UC(C_5) \neq 0$
- the body and the component are in relation at point $M(C_3)$ through a spring. A relation that links the displacement and the force is imposed at this point.
- the point $M(C_6)$ is a free point, the efforts are null for this point.

These characteristics constitute the boundary conditions of the problem.

The solution of the problem gives forces and displacements at the points of the components. The figure 5 shows displacements due to the deformation of the component through imposed displacements or forces.
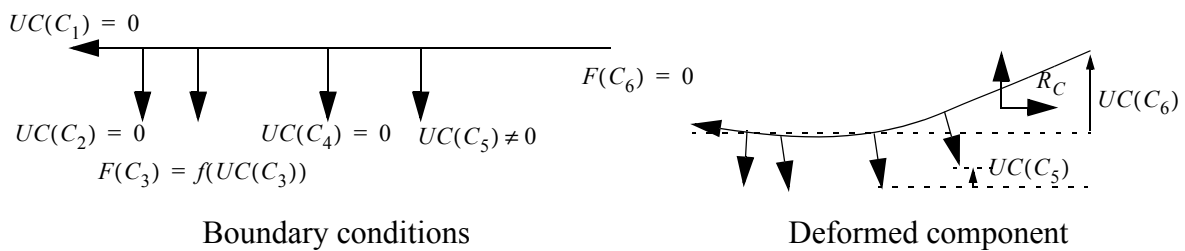

Figure 5: Variation due to the flexibility and to external forces

4   Taking into account displacements and geometric deviations simultanously

The study of the geometric tolerance of rigid components makes it possible to know the relative situation of the various components of the system, and more particularly the variations of position of the points authorised by geometrical tolerances. When components are compliant, variations of geometry due to manufacturing still exist. The variation of the manufactured geometry is defined in its free state. The difference is that the compliance permits all the contacts

between the components.

Work on compliant part assemblies considers the deformation of components during assembly. These works are based on numerical simulation of assembly. Results give the influence of the component deviation on the final assembly and integrate the variation of the fixture position during the assembly stage [3], [5], [7], [8]. Samper and Giordano present in [6] two methods to achieve the tolerance analysis of a compliant system, that include the compliance of the component and the seal. The boundary conditions do not depend on a symbolic form of link parameters.

Our work lies on assembly for which none of displacement due to either compliance or geometric variations is negligible. The particularity of our work is that the boundary conditions are function of the geometric variation of components and links. When the components are in contact, the points of the components are the same, which is possible since components are compliant.

The figure 6 illustrates that compliance allows contacts at each link points.
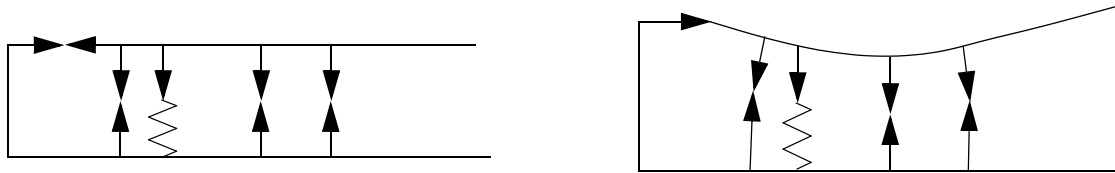

Figure 6: Global point position variation

4.1   Rigid component displacement

Previously exposed §2.1, displacement of components' point after rigid body movement are found. This movement is due to links' position variation. The difference between the nominal position and the position after rigid body movement is called UD.

For each point of the component, UD is the sum of products between influence coefficient of the link in one direction, $IC1_j$ and the variation of the link position in this direction, UMj.

$$UD_i = \sum_j IC1_j \cdot UM_j \qquad (2)$$

4.2   Boundary conditions

In a typical point of view, boundary conditions may be either in force or in displacement or both but they don't depend on parameters. In a matter of fact, links' position variation UM_j influence boundary conditions. In our work, UM_j are parameters of the boundary conditions.

4.3   Results

Solving boundary condition equations gives UC variation for all the components' points with links' position as parameters. Results can be read as the sum of two elements: a variation of position called Cst corresponding to components flexibility when link's position variation are all null and another one due to this variations. This second element can be read as the sum of products between the influence coefficients of the link in one direction $IC2_j$ and the variation

of the link position in this direction UM_j.

$$UC_i = Cst_i + \sum_j IC2_j \cdot UM_j \tag{3}$$

The interesting variation is the total variation UT. In this case, UT is the sum of two deviations UD and UC.

$$UT_i = UD_i + UC_i = Cst_i + \sum_j (IC1_j + IC2_j) \cdot UM_j \tag{4}$$

In this last equation, one can see that the last point position is given according to links' position. This result integrates the effects of geometric variations and of the compliance.

5 Example

5.1 Presentation

The study of the gaps between the body car (B) and the door (D) of a vehicle is proposed.

The position of side door points and body points are presented figure 7. Influence of links' position on door's point position is studied in order to determine gap and flush between these two components.
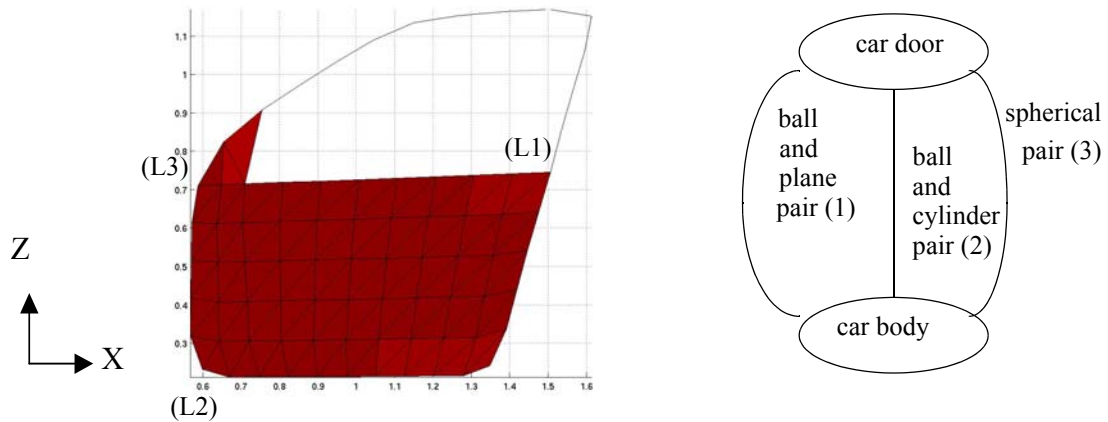

Figure 7: Example Geometrical Pairs Data

Design models and stiffness of components are imported from finite element modeling. Geometrical pairs are given.

5.2 Model

Links between the two components are located on three points: two hinges and a lock. The models of the links are a ball and plane pair (1), a spherical pair (2) and a ball and cylinder pair (3), shown in figure 7. These connections generate a non-over constrained view of the mechanism. Strain comes from forces due to seal located all around the door. The seal is modeled by springs between each opposite points of the two components (figure 8). These points are located over nodes defined by the finite element model all around the door geometry. Equivalent stiff-

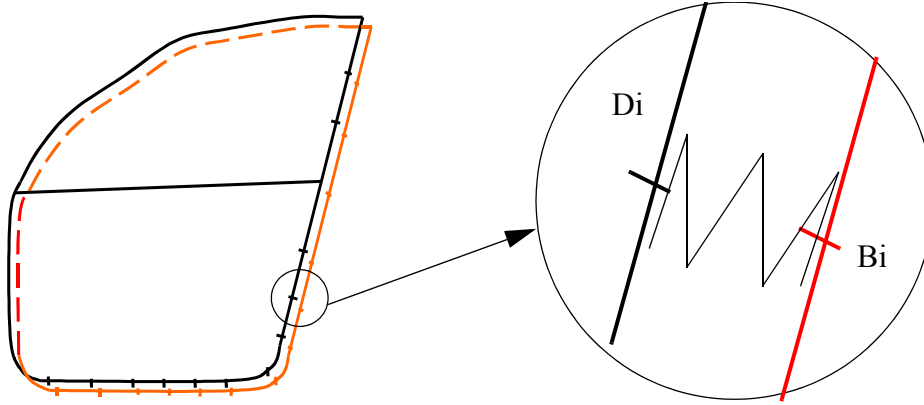ness of springs should be taken as close as seal's one.

Figure 8: Position of Body car compared to the door one

Body car is supposed to be rigid for this example, then door point positions are real interesting points. Parameters are variations of positions of link points.

Three types of boundary conditions are imposed:

- variation of link point positions due to compliance is null. In fact it comes from the fact that our mechanism model is not over constrained. It gives us 6 boundary conditions.
- variation of seal point positions due to compliance is linked to force at this point:
- force in all other points is null. Whatever the direction, if there is no link, no specific force and no specific displacement imposed to a point, force at this point is null

If one consider a spring, the usual relation between the displacement and the force is the following: F = K Δu, where K is the stiffness of the spring, F the force and Δu the variation of length of the spring. In our case, spring direction is taken as Y. For each point where seal is considered, equation 5 is written:

$$F_y[i] = K_i \Delta u[i] \tag{5}$$

This Δu[i] have to be expressed with our variations. Δu[i] represents the variation between the non-loaded length of the spring called L_{init} and the loaded one. Length corresponding to the loaded length in our model, is the difference $T(B)T(D)$ for points associated with the seal (figure 9).
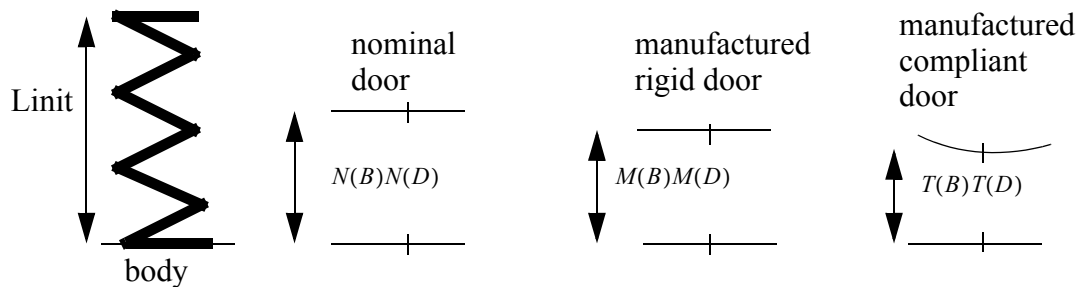

Figure 9: Seal Model

The equation 5 gives the following equation:

$$F_y[i] = K_i(L_{init} - (UT_y[D_i] - UT_y[B_i])) \qquad (6)$$

In our case, the body car is a rigid element, and it is shown that it is possible to explain $UT[i]$ for each component in a symbolic way with link variations of positions as parameters. Then, equation (6) gives a second type of boundary condition such as:

$$F_y[i] + K_iUC[D_i] = K_i(1, 512 + 0, 125\,UM[D_1] + 0, 326(UM[D_3])) \qquad (7)$$

The right side of equation (7) written in symbolic form takes place in our system as a symbolic matrix called SBC. If the unknown matrix is considered, build with displacements and forces:

$$[unknowns] = \begin{bmatrix} Displacements \\ Forces \end{bmatrix} \qquad (8)$$

Thanks to these three types of boundary conditions, the following system is written:

$$\begin{bmatrix} boundary condition coef \end{bmatrix} \begin{bmatrix} unknowns \end{bmatrix} = \begin{bmatrix} SBC \end{bmatrix} \qquad (9)$$

In this example, 88 points are considered, with 6 degrees of freedom for each of them. This provides 88*6=528 equations.

Completing with the following system:

$$\begin{bmatrix} F \end{bmatrix} = \begin{bmatrix} K \end{bmatrix} \begin{bmatrix} Uc \end{bmatrix} \Rightarrow \begin{bmatrix} K \end{bmatrix} \begin{bmatrix} Uc \end{bmatrix} - \begin{bmatrix} F \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \begin{bmatrix} K \end{bmatrix} & \begin{bmatrix} -Id \end{bmatrix} \end{bmatrix} \begin{bmatrix} unknowns \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \qquad (10)$$

At final, a system made by 2*528=1056 equations is to be solved, where [SBC] is a symbolic matrix the coefficients of which depend on link variation positions, [unknowns] represents compliance displacements and forces on each point.

$$\begin{bmatrix} \begin{bmatrix} K \end{bmatrix} & \begin{bmatrix} -Id \end{bmatrix} \\ [boundary conditions] \end{bmatrix} \begin{bmatrix} unknowns \end{bmatrix} = \begin{bmatrix} SBC \end{bmatrix} \qquad (11)$$

Solving this system leads to symbolic results giving point positions as functions of link situation parameters such as presented (equation 12).

$$UC_y[D_6] = -0, 003 + 0, 0225\,UM_y[D_1] + 0, 0225\,UM_y[D_2]$$
$$UD_y[D_6] = -2, 007\,UM_y[D_1] - 2, 007\,UM_y[D_2] \qquad (12)$$
$$UT_y[D_6] = -0, 003 - 1, 984\,UM_y[D_1] - (1, 984\,UM_y[D_2])$$

The position variation of point 6 belonging to the door in the Y direction due to compliance, called $UC_y[D_6]$, can be given as a function of connection parameters, called $UM_y[D_i]$. These parameters are variations of position of connection in one direction, compared to the nominal one. The first numerical element of the result gives the variation of position due to compliance,

without any variation of position of links. Coefficients given before each variation of position of links are called influence coefficients and show importance of this variation on the point position. It both takes into account the rigid body movement and the component compliance.

Coefficients of influence of each links position in UT are different from the coefficients of the same link in UD. That means that the final result is not a sum of two simple variations but a more complete term, where boundary conditions are symbolic and depend on link position variations.

If numerical value is affected at each link position, each point position is to be found and thus general component figure is obtained. Figures presented below show results for a specific value of the link position variation: the ball and plane pair and the spherical pair have no variation; the ball and cylinder pair has a variation of position of 0.1 mm in the Y direction. Either the position after the rigid body movement, figure 10, or the variation of position due to compliance, figure 11, or the global variation, figure 12, is to be found.
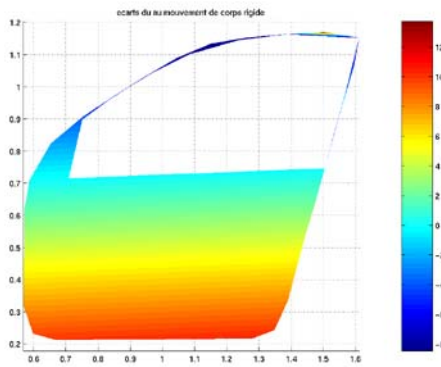

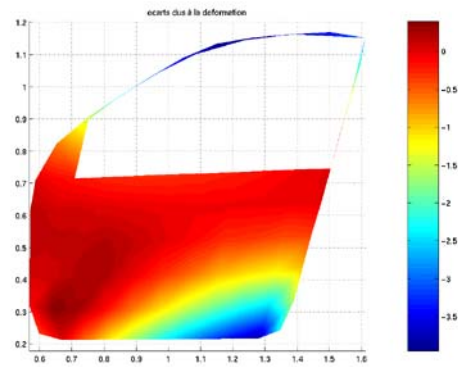
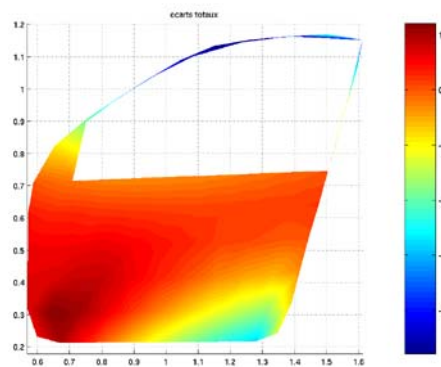*Figure 10: Rigid Body Movement*   *Figure 11: Compliance influence*

Figure 12: Total variation

6   Conclusion

The study gives the general steps which makes it possible to bind the geometrical variations of the components and the deformations. The illustration of steps is restricted with a simple and a sufficient case to illustrate our matter. The object of our work consists in undertaking this

study on three-dimensional cases, while benefiting from our experiment of the tolerance of the rigid components and the results of calculations of deformations. This work of integration of the deformations in the studies of tolerance constitutes a step towards the taking into account of all the influential parameters on the geometry of the mechanical system such as the variations of manufacture, the rigidity of the components, the temperature...

References

[1] E. BALLOT, *"Lois de comportement géométrique des mécanismes pour le tolérancement"* , PhD thesis, LURPA, Université de Nancy I, 1995

[2] P. BOURDET, L. MATHIEU, C. LARTIGUE, and A. BALLU, *"The concept of the small displacement torsor in metrology"*, Proceedings of the International Euroconference on Advanced Mathematical Tools in Metrology, 1995

[3] J. CAMELIO, S.J. HU, D.J. CEGLAREK, "Modeling variation propagation of multi station assembly systems with compliant parts", Proceeding of DETC'01, ASME 2001, Pittsburg, Pennsylvania, September 9-12, 2001

[4] J.Y. DANTAN ,F. THIEBAUT ,A. BALLU, P BOURDET., *"Functional and manufacturing specifications - Part 1&2"* , Kluwer Academic Publishers - Congrès IDMME'2000 , 2002

[5] K.G. MERKLEY, *"Tolerance analysis of compliant assemblies"*, Ph.D. Thesis, Department of Mechanical Engineering, Brigham Young University, 1998

[6] S. SAMPER, M. GIORDANO, *"Simultaneous analysis method for tolerancing flexible mechanisms"*, Proceedings of the 7th CIRP International Seminar on Computer Aided Tolerancing, April 24-25, 2001, Cachan, France, pp 75-82, 2001

[7] E. SELLEM C. A. de HILLERIN A. CLEMENT A. RIVIERE, *"Modelling-Simulation-Testing of Compliant Assemblies"*, Global consistency of tolerance, proceedings of the 6th International Seminar on Computer Aided Tolerancing, University of Twente, Enschede, The Netherlands, 22-24 March 1999, Kluwer Academic Publishers, pp 355--364, 1999

[8] B. SHIU, D.W APLEY, D. CEGLAREK, J. SHI, *"Tolerance allocation for compliant beam structure assemblies"*, IIE Transactions, n°35, pp 329-342, 2003.

[9] F. THIEBAUT, P. BOURDET, "*Geometric model of mechanical links : application to tolerance analysis"*, Tolerance Modeling and Analysis, ASPE Summer Topical meeting, Charlotte (North Carolina, USA), pp. 38-43, july , 2002

[10] F. THIEBAUT, *"Contribution à la définition d'un moyen unifié de gestion de la géométrie réaliste basé sur le calcul des lois de comportement des mécanismes"*, Ph.D. Thesis, Laboratoire Universitaire de Recherche en Production Automatisée, Ecole Normale Supérieure de Cachan, 2001