

DAC-21023

AN INTEGRATED FRAMEWORK FOR 3D TOLERANCE CHAINS IN DESIGN AND MANUFACTURING

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ABSTRACT

This paper is about a new model of geometric deviation to compute tridimensional tolerance chains. By the use of a simplification of geometric deviation well adapted to computation of tolerances, we will show how to compute systematically the consequences of geometric deviation from functional requirements to manufacturing. Under the hypothesis that geometric deviations lead only to small displacement of parts, we will demonstrate how their propagation can be computed. We will highlight the improvement yielded by the proposed model and the consequences on the description of tolerances and manufacturing.

Keywords : *tolerance, 3D tolerance chain, machining*

INTRODUCTION

Background and motivation

Computation of tri-dimension tolerancing chains is a long, difficult and costly process for firms. The ever increasing demand for quality is a constraint that forces the mechanical industry to undertake the search and development of new tools of help for this complex task [22].

The following model aims to contribute to bring a matter to this issue in two ways, firstly by the automation of the calculation process and secondly by the improvement of its precision by taking into account the tri-dimensions of the geometry in the deviation model [23].

The process of tolerancing and the tolerance chain computation is not only the concern of designers but also involves manufacturers and quality inspectors. In order to take this into account we will address the assembly representation as

well as the manufacturing process description in the same model and with the same concepts.

Several models already exist in this field of research, however by our review of the literature, we will show that there is no approach that allows a global dealing of this problem from design to inspection.

Literature review

In a classical approach to the specification of tolerances on mechanical parts, the expression of constraints is reduced to a one-direction problem by the projection of the tolerance domains along projection axes, even if, in recent years, numerous works have added to the field [2,15, 21]. Thus, for every main direction of the part, a set of simplified constraints named dimension chains is obtained: a vector represents each dimension. This very simplified geometric model allows the designer to split the tolerance values between the elements of the dimension chains. This methodology is specially used in the application of tolerance chains to manufacturing process. However, recent work takes surface rotation into account as geometric deviation.

Description of tolerance at the design stage evolves with standard and with new research approaches [20,24]. A lot of work has been carried out in the past ten years to develop a mathematical formulation of tolerances. Two main approaches can be distinguished: by space allowed around theoretic geometry [16] or by parameterization of deviation from theoretic geometry [8,11,17,19]. The first approach seems more realistic to describe tolerance standards as the second is closed by measuring machine and gives computation opportunities. The proposed approach is of the last type.

Numerous algorithms are now available to split tolerance values between parts in a mechanism or during manufacturing

[2]. This problem is also known as tolerance synthesis. One can find, worst case algorithms or statistical approaches and the computation are made by optimization algorithms or simulation [1,10,12,26].

The constant changes of the models between the simulation of the mechanism that makes use of one-direction dimension chains, the standardized specification by datum and tolerance zones, and the tridimensional metrology, yield a reduction of the tolerances. And then bring conflicts within the interpretation and the transfer of data from one model to the next.

We could not find in the review of the literature a tolerance model that gives answers to the various problems of the tolerancing process, i.e. a model that could be used from the expression of the functional requirements to the manufacturing and the control process. The aim of this paper is to describe a model for the whole process as a contribution to tri-dimensional chains and to make up for the lack of integration between models.

MECHANISM AND MANUFACTURING DESCRIPTION

To introduce the proposed model of geometric deviations and associated principles of computation, an example will be presented in parallel of theoretical considerations. This example will not be entirely given but we will focus on a few extracts.

The mechanism used is represented by figure 1. Even if the proper functioning of this small assembly requires the checking of numerous functional requirements, in this paper, we take only two into account. These two assembly conditions are expressed by gaps between surfaces B1 and P1 for the first functional requirement and between surfaces P2 and A2 for the second functional requirement.

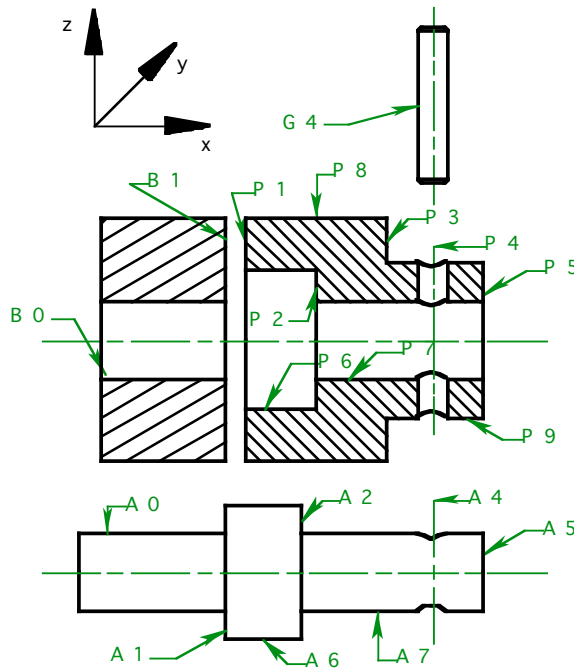


Figure 1: example mechanism

First, we will introduce the mathematical concepts of the geometric deviation model before saying how it can be applied to an assembly or to the machining process of parts.

Mainly, we will give the principles of a formal computation for the complete set of deviation propagation by expressing the small displacement of every part in a mechanism as a function of geometric deviation. An application of this methodology to tolerance chains will be presented in the next section.

Proposed deviation model : the small displacement torsor

A small displacement torsor is the formalization of a displacement for which a limited development to the first order of the rotation is used [7]. A small displacement torsor hence accurately describes the small displacement of a solid or of a surface between two positions, which are close to each other. That is the case for most tolerance problems.

A small displacement torsor T is then described by the combination of two vectors: a rotation vector \vec{R} called resultant and a displacement vector $\vec{D}(O)$ called moment and expressed at a point O . A small displacement torsor is thus noted:

$$T = \left\{ \vec{R} \quad \vec{D}(O) \right\}$$

Expression from one point to another is:

$$\vec{D}(A) = \vec{D}(O) + \vec{R} \wedge \vec{OA} \quad (1)$$

Further on, we will use the following notation with components of both vectors:

$$T = \left\{ \begin{matrix} \alpha & u \\ \beta & v \\ \gamma & w \end{matrix} \right\}_O \quad (2)$$

Small displacement torsors will be used to describe all geometric deviations or displacements resulting from real parts geometry. This includes:

- the deviation between the nominal surface and the substitute surface (representation of the real surface),
- the gap between two surfaces of a link,
- the displacement of a part in a mechanism from the nominal position,
- the envelop surface of a machining tool,...

However some components of a deviation torsor have no impact for tolerances. These are the components that leave the theoretical surface unchanged by the small displacement or cinematic degree of freedom for gaps. These components are usually set to null in metrology when small displacement torsor are used. These components will be marked by I for surface deviation and by Ind for gap. This notation is followed by indications of the type of component: rotation or translation and axis in the local frame.

Figure 2 shows the shape of a deviation torsor for a flat surface. It is an original part of this work that will be helpful to compute deviation spreading in the mechanism as well as over

constrained degrees of freedom. Section 3 will give details about it.

Mechanism application

This approach is built with a systematic association of a deviation model to every geometric element of a mechanism.

Deviation Torsor

This torsor represents the deviation from the theoretical surface to the substitute surface. The substitute surface is the description of the real one in the model. The parameterization associated to the substitute surface allows for the description of the whole set of machining surfaces. Figure 2 shows the different surfaces and parameters between theoretical and substitute surfaces. The association criterion between the substitute and the real surface is usual and will not be described here.

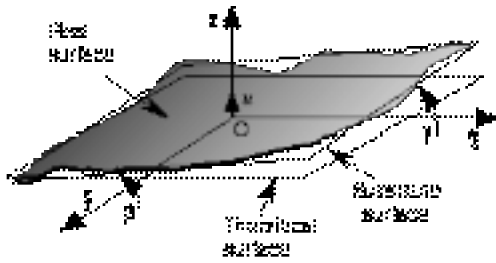


Figure 2: deviation torsor for a Plane

Equation 3 describes the form of the deviation torsor for a plane surface as illustrated by figure 2. This torsor is expressed in the local reference of the theoretical surface.

$$T_{S/P} = \begin{Bmatrix} I_{rx} & u \\ \beta & I_{ly} \\ \gamma & I_{lz} \end{Bmatrix}_O \quad (3)$$

The form of deviation torsor associated to usual surfaces can be found in [3,4].

For a given part substitute surfaces can be identified by a coordinate measuring machine by an association criterion to the set of measured points [6]. In this model, parameter values of the deviation torsor express machining deviations for the part.

Gap torsor

A gap torsor represents the gap between two substitute surfaces of different parts which are nominally in contact. There will be a gap torsor associated to each couple of surfaces that belong to different parts and are in contact.

The construction of the gap torsor relies on the application of the principles of calculation of propagation of deviations that we described in the section above. Details of this calculation can be found in papers. In the case of a cylinder with an axis parallel to \vec{y} on a plane with an axis parallel to \vec{z} one gets the following gap torsor shape:

$$T_{P/C} = \begin{Bmatrix} J_{rx}(P,C) & Ind_{tx}(P,C) \\ Ind_{ry}(P,C) & Ind_{ty}(P,C) \\ Ind_{rz}(P,C) & J_{tz}(P,C) \end{Bmatrix}_O \quad (4)$$

where J_{td} represents the type t ones for a translation or r gap components for a rotation and the d ones for the axis. Similarly, one gets Ind_{td} as undetermined gap components with the same conventions. These undetermined components are variables that represent sliding or unknown small rotations between parts. These variables are useful to calculate the positions of parts by classifying components according to whether they can transmit geometric errors (position) or cannot (cinematic degree of freedom). Due to the parametric description of the contact point this avoids discontinuities that occur when the point is preliminarily fixed.

Part torsor

A part torsor is associated to each part of a mechanism. It describes the displacement of a frame associated to the theoretical part under deviations coming from gaps and deviations. Displacement parameters of each part are unknowns that have to be expressed as functions of gaps and deviations by the methodology proposed in section 3.

One of the part is arbitrarily chosen as the reference and its part torsor is set to null.

Functional requirement

A k functional requirement f_k is expressed here as a geometric constraint of maximum or minimum distance or orientation between two substitute surfaces. This expression takes place on border points of the tolerance zone, according to the linearity of the small displacement. To do so, components of torsors are "projected" by a comoment operator [4].

$$f_k = (T_{1/A} + T_{A/R}^* - (T_{2/B} + T_{B/R}^*)) \cdot T_k \quad (5)$$

Comoment:

$T_1 \cdot T_2 = \alpha_1 \cdot u_2 + \beta_1 \cdot v_2 + \gamma_1 \cdot w_2 + u_1 \cdot \alpha_2 + v_1 \cdot \beta_2 + w_1 \cdot \gamma_2$ and where T^* is the torsor of the small displacement of the part under deviations. The computation of T^* will be described in the next section and T_k describes the direction of the translation or rotation required to express the functional requirement at a given point of the domain.

In the model proposed, a functional requirement is described by a function of parameters of deviation from several parts: the tolerance chain.

The tolerance model

In order to give a tolerance specification on each part involved in the chain, the deviation parameters will be determined per part. The following equations shows this principle.

$$\begin{aligned}
f_k(\alpha_{i,p}, \beta_{i,p}, \gamma_{i,p}, u_{i,p}, v_{i,p}, w_{i,p}, \mathbf{dr}_{i,p}) &\leq c_k \\
i \in \text{surfaces}, p \in \text{parts} \\
\left\{ \begin{array}{l} f_{k,1}(\alpha_{i,1}, \beta_{i,1}, \dots, \mathbf{dr}_{i,1}) \leq t_{k,1} \\ \vdots \\ f_{k,p}(\alpha_{i,p}, \beta_{i,p}, \dots, \mathbf{dr}_{i,p}) \leq t_{k,p} \end{array} \right. & \quad (6) \\
\text{and } \sum_p t_{k,p} &\leq c_k
\end{aligned}$$

Where $t_{k,i}$ are values of tolerances. The computation of these values is a well known formulated problem named tolerance synthesis. Various sharing methods exist and are summarized in surveys [14,18].

Each constraint $f_{k,p}$ is expressed on limit points of the tolerance zone. The set of tolerances on a part is then described by a set of linear constraints: the tolerance domain. In few cases the can be illustrated with mathematica [9]

The following figure 4 gives an example of a tolerance domain resulting from the whole tolerancing process with this model. In this case, the domain represents a distance requirement between two parallel flat surfaces with normal parallel to \vec{x} axis and bound by cylinders as A1 and A2 as represented by figure 3.

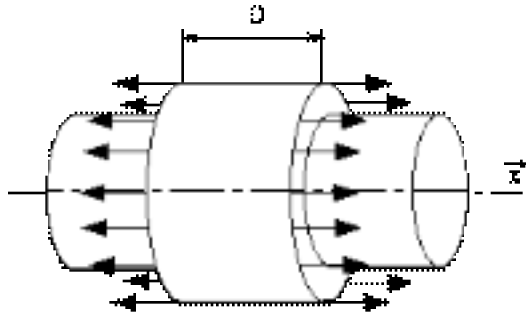


Figure 3 : expression on limit points of surfaces

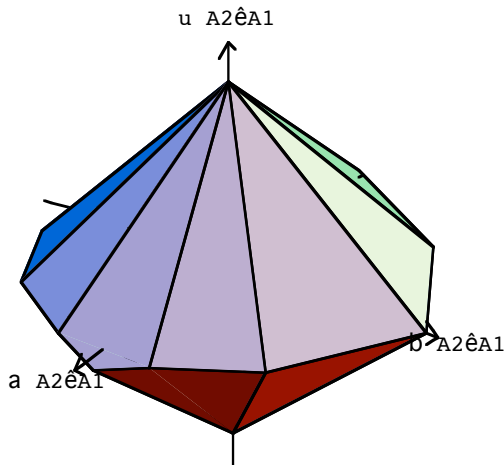


Figure 4 : illustration of the tolerance domain resulting from functional requirement on distance D

COMPUTATION OF TOLERANCE CHAINS

We have briefly described data of this model. We will now focus on the computation of tolerance chains methodology.

The part assembly in a mechanism is described by a graph where each circle is a part and each line is a surface and two parallel lines are a link. Different models found in the literature inspire this graph [13]. Figure 5 represents the graph for the mechanism proposed in figure 1.

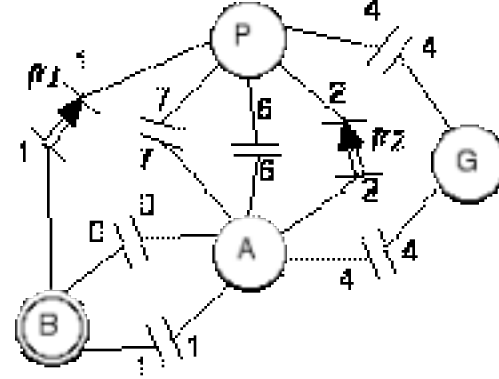


Figure 5 : graph of links with two functional requirements

The computation of the propagation of geometric gaps within the mechanism will be carried out in two stages. First, we will show how the gaps spread via a link and then how these propagations can be unified when a part is assembled in a mechanism via several links.

Propagation of deviation from one link

The propagation of deviations from a link i between surface y of part B and surface x of part A is computed directly from the composition of torsors from one part to another. The composition also includes the gap torsor between substitute surfaces x and y as the following formula shows. All these computations, of part B deviation from link i , $T_{B/R}^{(i)}$, are made in a common reference frame R .

$$T_{A/R}^{(i)} = T_{B/R} + T_{y/B} - T_{x/A} + T_{x/y} \quad (7)$$

There are as many expressions of torsor compositions as there are links in the mechanism. For part G , one gets the following expression:

$$T_{G/R}^{(1)} = T_{4/A} - T_{4/G} + T_{4/4} \quad (8)$$

$$T_{G/R} = \begin{cases} \alpha_{A/R} + \alpha_{A4/A} - \alpha_{G4/G} + J_{r4x} \\ \beta_{A/R} + \beta_{A4/A} - \beta_{G4/G} + J_{r4y} \\ \gamma_{A/R} + \text{Ind}_{r4z} \\ -(\mathbf{dr}_{A4} + \mathbf{dr}_{G4})\cos(\theta_4) - z_4 J_{r4y} + J_{t4x} + u_{A4/R} - u_{G4/G} \\ z_4 J_{r4x} + J_{t4y} - (\mathbf{dr}_{A4} + \mathbf{dr}_{G4})\sin(\theta_4) + v_{A4/R} - v_{G4/G} \\ \text{Ind}_{t4z} + w_{A/R} \end{cases} \quad \text{where } \begin{matrix} \text{where} \\ 0 \end{matrix}$$

e z_4 and θ_4 are contact point parameters in the link 4/4.

Part general assembly procedure

Usually, a part of a mechanism is positioned by several links in parallel. For a part, one gets different expressions given by applying the previous formula to each link. Nevertheless all these expressions partially describe the same position of the part. Each link gives a partial position because cinematic degrees of freedom, noted Ind_{ji} , usually remain. So the next formula defines the position of a part p by $T_{P/R}^*$ as a set of $l-1$ torsor equalities.

$$T_{P/R}^* \equiv T_{P/R}^{(1)} = \dots = T_{P/R}^{(i)} = \dots = T_{P/R}^{(l)} \quad (9)$$

For the mechanism, one gets, for instance:

$$\begin{aligned} T_{A/R}^* &\equiv T_{A/R}^{(0)} = T_{A/R}^{(1)} \\ T_{G/R}^* &\equiv T_{G/R}^{(4)} \\ T_{P/R}^* &\equiv T_{P/R}^{(4)} = T_{P/R}^{(6)} = T_{P/R}^{(7)} \end{aligned} \quad (10)$$

This system of equations contains many local degrees of freedom (i.e. related to a single link) that could be suppressed by another link. In order to compute the complete position of a part, we try to solve this system of n undetermined Ind_{ji} and $m=6.(l-1)$ lines. The result given by the pivot algorithm is as follows:

$$\left. \begin{aligned} a_{11}Ind_{j1} + a_{12}Ind_{j2} + \dots + a_{1n}Ind_{jn} &= b_1 \\ c_{22}Ind_{j2} + \dots + c_{2n}Ind_{jn} &= \tilde{b}_2 \\ &\vdots \\ k_{rr}Ind_{jr} + \dots + k_{rn}Ind_{jn} &= \tilde{b}_r \\ 0 &= \tilde{b}_{r+1} \\ &\vdots \\ 0 &= \tilde{b}_m \end{aligned} \right\} \quad (1)$$

$$\left. \begin{aligned} 0 &= \tilde{b}_{r+1} \\ &\vdots \\ 0 &= \tilde{b}_m \end{aligned} \right\} \quad (2)$$

The first result of this system (1) is the expression of some local undetermined cinematic degrees of freedom as gaps according to other links and rank r of the system. The second result is a set of conditions (2) between deviation parameters and gap parameters that have to be respected. From a designer's point of view, it means that there is a set of valid assembly and working configurations for the mechanism. This set is described by every right choice of setting gaps parameters. This system is also useful in building an assistant for the designer to analyze mating behaviors [10,12].

For instance, the positioning of the shaft A according to the structure B (figure 1) gives the following compatibility system.

$$\begin{cases} \beta_{A0/A} + \beta_{A1/A} - \beta_{B0/B} + \beta_{B1/B} - J_{r1y} + J_{r2y} = 0 \\ \gamma_{A0/A} - \gamma_{A1/A} - \gamma_{B0/B} + \gamma_{B1/B} - J_{r1z} + J_{r2z} = 0 \end{cases}$$

This example shows that if the designer selects a flat mating condition then $J_{r1y} = J_{r1z} = 0$. The others gaps can now be

computed as a function of geometric deviations and cannot be freely set, as parts are considered rigid.

We have explained the principles and a computation procedure of deviation spreading in a mechanism, a tri-dimensional tolerance chain. The next section applies these principles to machining.

Manufacturing as an assembly

The manufacturing of mechanical part is made with the help of machining supports, each dedicated to an operation. As a mechanism, it is also assemblies in a manufacturing frame. One previously machined surface makes an assembly with the support in order to machine new surfaces, etc.

The surface created by the machining is considered as the tool surface or tool envelope surface.

The integration of manufacturing data in the same framework as used at the design stage allows simulates the machining. Functional requirements values can now be shared between tolerances by taking several machining support and tool deviations into account.

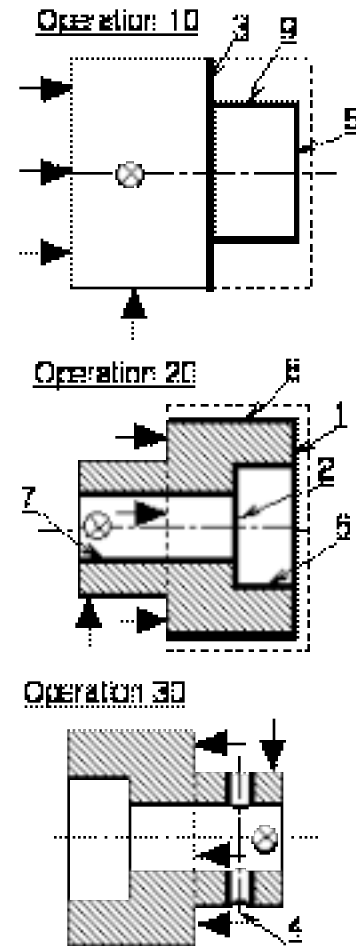


Figure 6: Machining process of the gear P

Figure 6 illustrates the machining process for the manufacturing of the gear. This machining process can be translated into an assembly graph for the computation of tolerance machining chains.

The next figure illustrates, with the same convention as used in figure 5, the machining of gear P according to the three operation processes given in figure 6. Graph

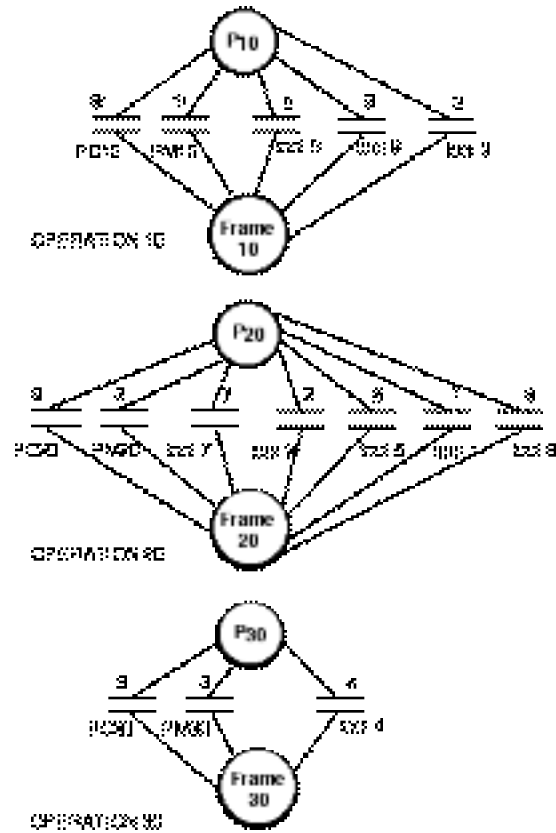


Figure 7: Graph of gear P machining process

The computation of the manufacturing process by principles explained in section 3 allows to express functional requirements as functions of deviation coming from machining supports, tools and multiple positioning.

The next equation illustrates this principle on the functional requirement f_{r2} between surfaces 2 of parts A and P, with machining parameters for part P. An abbreviation is used for tools: O .

$$\begin{aligned} & z_{p2} (\beta_{A2/A} - \beta_{A7/A} - \beta_{O2/R} + \beta_{O7/R}) + \\ & z_5 (\beta_{A7,A} - \beta_{A4,A} + \beta_{O4,R} - \beta_{O7,R} - \beta_{PM20,R} + \beta_{PM30,R}) + \\ & y_{p2} (-\gamma_{A2,A} + \gamma_{A7,A} + \gamma_{O2,R} - \gamma_{O7,R}) - \\ & (d_{G4} - d_{P4}) \cos(t_5) + (d_{P4} + d_{O4}) \cos(t) + u_{A2,A} - u_{A4,A} - \\ & u_{O2,R} + u_{O4,R} - u_{PM20,R} + u_{PM30,R} \end{aligned}$$

This last equation shows how a functional requirement could be systematically expressed as a function of deviations of tools and machining support.

CONCLUSION

We have proposed a methodology for the processing of tridimensional dimension chains. On the basis of a model of geometric defaults, it defines a computation of the propagation of these defaults. It was shown before that the processing of dimension chains in manufacturing can be formulated in a one an single way, which allows for the resolution and the integration of the results for a processing within current engineering of the methodology. The simulation of the manufacturing actually allows for a sharing of the tolerance values adapted to the means of production available

This methodology programmed under the Mathematica software [25] is presently coupled with a CAD software so as to directly gather the geometry of the parts, hence a fast and systematic processing of mechanisms.

The development of the models allows for he processing within a homogeneous frame of tolerancing, including metrology of parts [5].

However, new questions arise: how can the translation from the deviation parameters into tolerance standard be systematized? How can algorithms of tolerance sharing values take into account the shape of tolerance domain and machining deviation parameters?

Knowing the dispersions of manufacturing according to the gap parameters of the model seems to be an important basis for the research but also for a better knowledge of these phenomena of geometric dispersion

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