Controlling a Complex Surface with a 3 Axis Measuring Machine

(Original french version translated into english)

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SUMMARY : Controlling complex surfaces (surfaces with an algebraic equation of an order greater or equal than 2, or transcendantal equations) is highly difficult with the usual methods.
Since the 3 axis measuring machines appeared, the task of researchers has been to solve the two main aspects of this control:
- Positioning the object to be controlled (1) and (2).
- Evaluating the difference between the theoretical and actual forms, each surface being specifically studied.
These works, although they have certainly helped the manual process, have not, however, suppressed the result's subjectivity, as they leave to the operator the choice of the positioning of the control reference. This choice is of great influence upon the value of the obtained differences. The method suggested hereafter is perfectly objective it suppresses the accurate balancing of the object and provides with a reference such that differences are minimum.
This method can also be applied to the problem of balancing raw objects of huge dimensions.

Position of the problem:

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- a) Positioning the object to be controlled [1] and [2].
- b) Evaluating the difference between the theoretical and actual forms, each surface being specifically studied [3].

These works, although they have certainly helped the manual process, have not, however, suppressed the result's subjectivity, as they leave to the operator the choice of the positioning of the control reference. This choice is of great influence on the value of the obtained differences.
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This method can also be applied to the problem of balancing raw objects of huge dimensions (alternator casing, turbine shaft etc...

Presentation of the method:

Let us consider the surface (S) defined in fig. 1 to be controlled the «fitting engine». The surface (defined in the frame AX, Y, Z) is not necessarily known by its equation but can be defined by a list (X, Y, Z, a, b, c) of the coordinates X, Y, Z of a series of n theoretical points (in which the control will be applied) and by the components a, b, c of the normal vectors to the surface at the considered points.

If the part is positioned in a non-over constrained set-up on the CMM (Coordinate Measuring machine), the measured points obviously differ from their theoretical positions (fig.2), and some deviations can be observed.

Currently, the operator attempts to align the two sets of points, either manually or with the help of a computer which determines the necessary displacement to be performed by considering that the part is perfect (traditional axis change calculation). The piece is not perfect, the operator repeats this operation several times until he estimates the matching to be correct.

Despite the use of a computer, the operation is completely subjective and the results of control will vary in function of the operator.

In the paper, we propose to determine the optimal fitting between the measured and the theoretical points.
Let us consider $\begin{Bmatrix} \overrightarrow{\Omega_A} \\ \overrightarrow{D_A} \end{Bmatrix}$, the torsor of the small displacements, calculated at the point A, which should allow to move the part from its current position 1 to the position 2 so that the measured points best fit the theoretical ones in a coordinate frame attached to the measuring system.
Let us consider Xi, Yi, and Zi, the coordinates of the theoretical point Mth, and $\overrightarrow{n_i}$, the normal vector to the theoretical surface at the considered point (Fig. 3).

M_i is the actual point, defined as the intersection between the actual surface and $\overrightarrow{n_i}$.

As the actual surface does not perfectly match the theoretical one, the point M_i does not necessarily correspond to M_{th}, and the measured deviation along the normal vector $\overrightarrow{n_i}$ is given by $M_i\ M_{th} = \varsigma_i$. These deviations are minimized by giving a displacement so that the final position of the point M_i belongs to the tangent plane of the theoretical surface at M_{th}.

This can be expressed according to equation (1): $\overrightarrow{D_{Mi}} \cdot \overrightarrow{n_i} = \xi_i$ (1)

As the displacements are supposed to be very small, they verify:
$\overrightarrow{D_{Mi}} = \overrightarrow{D_A} + \overrightarrow{M_iA} \wedge \overrightarrow{\Omega}$

where A is a reference point.
Then:

$$(\overrightarrow{D_A} + \overrightarrow{M_iA} \wedge \overrightarrow{\Omega}) \cdot \overrightarrow{n_i} = \xi_i$$
$$\overrightarrow{D_A} \cdot \overrightarrow{n_i} + (\overrightarrow{M_iA}, \overrightarrow{\Omega}, \overrightarrow{n_i}) = \xi_i$$
$$\overrightarrow{D_A} \cdot \overrightarrow{n_i} + (\overrightarrow{AM_i} \wedge \overrightarrow{n_i}) \cdot \overrightarrow{\Omega} = \xi_i \qquad (2)$$

Expressed in this form, equation (2) represents the cross-product between the small displacement torsor, $\begin{Bmatrix} \overrightarrow{\Omega} \\ \overrightarrow{D_A} \end{Bmatrix}$ and the torsor of the *Plückériennes* coordinates of the vector $\overrightarrow{n_i}$,
$\mathcal{C}i \begin{Bmatrix} \overrightarrow{n_i} \\ \overrightarrow{AM_i} \wedge \overrightarrow{n_i} \end{Bmatrix}$
which can by written in the condensed form as: $\mathcal{C}i\, \mathcal{D} = \xi_i$

A similar relationship can be written for each measured point.
This leads to the linear system:

$$\boxed{\mathcal{C}^T \cdot \overrightarrow{\mathcal{D}} = \vec{\xi}}$$

where : $\vec{\xi}$ represents the vector of the measured deviations ξ_i
$\overrightarrow{\mathcal{D}}$ represents the vector of the displacement
$\mathcal{C}^T$ represents the transposed matrix of the plückériennes coordinates of the normal vector.
The unknowns are the 6 components (α, β, γ, u, v, w) of the small displacement torsor $\begin{Bmatrix} \overrightarrow{\Omega_A} \\ \overrightarrow{D_A} \end{Bmatrix}$.
As all the relationships in (2) are linear, 6 independent equations are necessary to solve the problem.
But, if only 6 points are measured, the number of measured points would not be sufficient to give a representative image of the part shape.

If we assume that the surface (or set of surfaces) is defined by n points (n = 100, for instance), it is possible to write n equations of type (I).
This gives a linear system of n equations for 6 unknowns.

The problem resolution thus consists in finding the small displacement that better satisfies the n equations. This can be solved by using the least-square method for which W, the residual defined by equation (3), must be minimized:

$$W = \sum_1^n (P_i \cdot \mathcal{D} - \xi_i)^2 \qquad (3)$$

The optimization problem leads to the following linear system:

$$\frac{\partial W}{\partial \alpha} = 0 \qquad \frac{\partial W}{\partial \beta} = 0 \qquad \frac{\partial W}{\partial \gamma} = 0$$

$$\frac{\partial W}{\partial u} = 0 \qquad \frac{\partial W}{\partial v} = 0 \qquad \frac{\partial W}{\partial w} = 0$$

The resolution of this linear system gives the components of the displacement torsor (α, β, γ, u, v, w), displacement that we call the *significant* displacement and which corresponds to the optimal balancing of the part.
It is interesting to notice that the deviation distribution is gaussian. As a result, ouliers do not significantly affect the final solution.

RESULTS:

The exploitation of the results is immediate:

1 – The *significant* displacement is applied to the measured points by calculation, and the new deviations between the theoretical shape and the actual one can be evaluated (distance to the tangent plane).
If a plotter is coupled to the CMM, it is possible to draw the surface topography of each section.

Results for the fitting engine are proposed below (extract of the results).

THEORETICAL DEFINITION OF THE PART

N°	X	Y	Z	Measured deviation Z axis	Actual deviation Z axis	
4	−77 389	76 189	0.0	2 200	0 000	
5	−32 129	0 0	− 38 099	1 400	− 0 202	
6	−73 709	0 0	0 0	2 290	0 150	
7	−32 250	12 699	− 38 099	1 490	− 0 256	HORS TOLERANCE
8	−73 809	12 699	0 0	2 360	0 054	
9	−32 639	25 399	− 38 099	1 490	− 0 332	HORS TOLERANCE
10	−74 129	25 399	0 0	2 290	0 048	
11	−34 159	50 799	− 38 089	1 520	− 0 268	HORS TOLERANCE
12	−75 369	50 799	0 0	2 220	0 006	
13	−36 659	76 199	− 38 099	1 500	− 0 210	
24	−66 399	228 599	− 38 109	1 170	0 277	HORS TOLERANCE
25	−29 219	254 000	− 38 899	0 580	0 271	HORS TOLERANCE
26	−61 459	254 000	− 50 799	1 020	0 204	
27	−27 189	279 399	−101 599	0 400	0 247	
28	−57 459	279 399	− 63 500	0 760	0 211	
29	−26 069	304 799	−114 299	0 410	0 334	HORS TOLERANCE
30	−54 259	304 799	− 76 199	0 610	0 028	
31	−25 619	330 199	−127 000	0 310	0 311	HORS TOLERANCE
32	−51 699	330 199	− 88 899	0 650	0 045	
33	−18 309	355 599	−152 399	0 230	0 098	
40	−28 469	419 099	−165 099	−0 019	0 008	
41	−48 239	419 099	−127 000	0 130	0 241	
42	−43 549	431 799	−139 699	−0 009	0 092	
43	−14 379	406 409	−190 500	−0 039	0 065	
44	−25 199	431 809	−177 789	0 010	−0 537	HORS TOLERANCE

It worth noting that the measured deviations along z introduced in the computer are better than the best results of control obtained by a companion after a huge effort to manually balance the part on the CMM. The reduction of the observed deviations is significant. In practice, it is obviously vain to attempt to manually carry out the best balancing.

2- The method assessment is performed as follows:
Considering a set-up of control defined by 6 adjustable contact points, a first series of measurements is carried out. From these measurements, the significant displacement of the part is calculated thanks to the proposed method. From the significant displacement, it is thus possible to calculate the displacement to be applied to each contact point Ci of the set-up according to the following equation:

$$\overrightarrow{D_{Ci}} = \overrightarrow{D_A} + \overrightarrow{C_iA} \wedge \vec{\Omega}$$

A new series of measurements allow the comparison between the optimized deviations and the new measured deviations. Results are consistent to the hundredth of a millimeter.

3- The experimental assessment previously described is technically (actually) a balancing operation. The method can thus be used to optimally solve any kind of balancing problem.

4- This method is very efficient for simple cases (see figures): flatness, circularity or cylindricity evaluations.

[1] SULZER G. – M. HOLLER : Zahnradmessung mit numerish geführter 3 – Koordinaten – Ne b maschine VDI – Z 116 (1974) Nr 14 SEITE : 1161 – 1161.

[2] MATTHIAS E. Lage und bewegung in werkzevgmaschinen ban
Fertigung 3 (1974) p. 87 – 90
Fertigung 5 (1974) p. 169 – 175

[3] WECK M – BAGM P : CIRP Annals (1975) p. 375 – 376.

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GEOMETRIC CONTROL OF A FITTING ENGINE

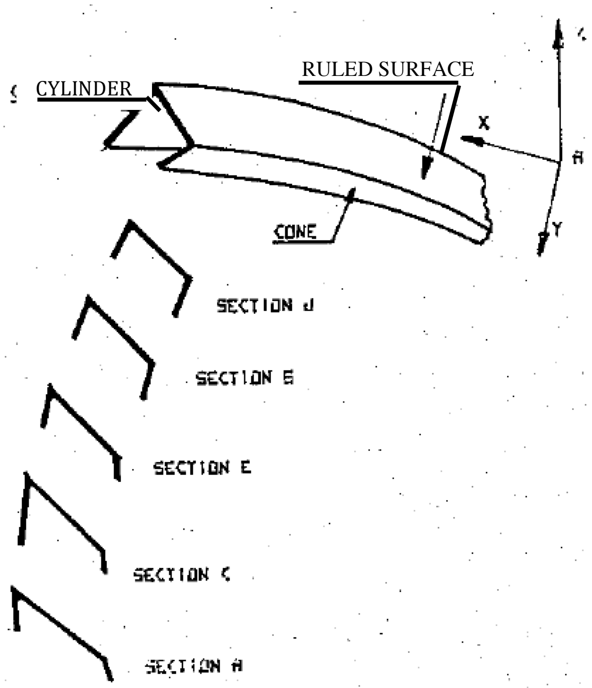

FIGURE 1

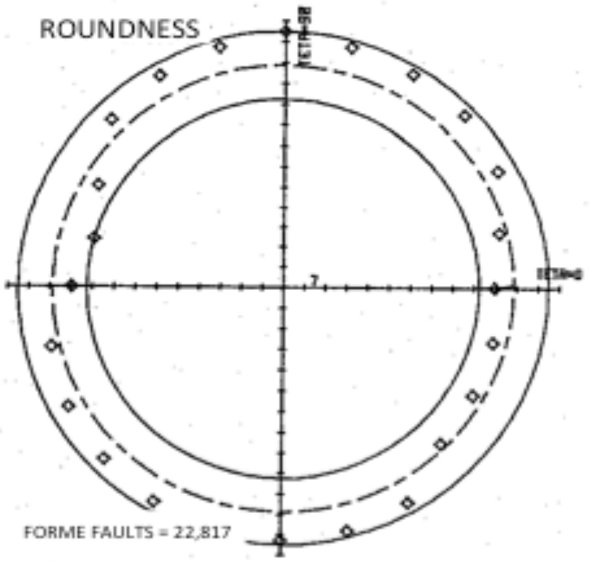

FIGURE 2

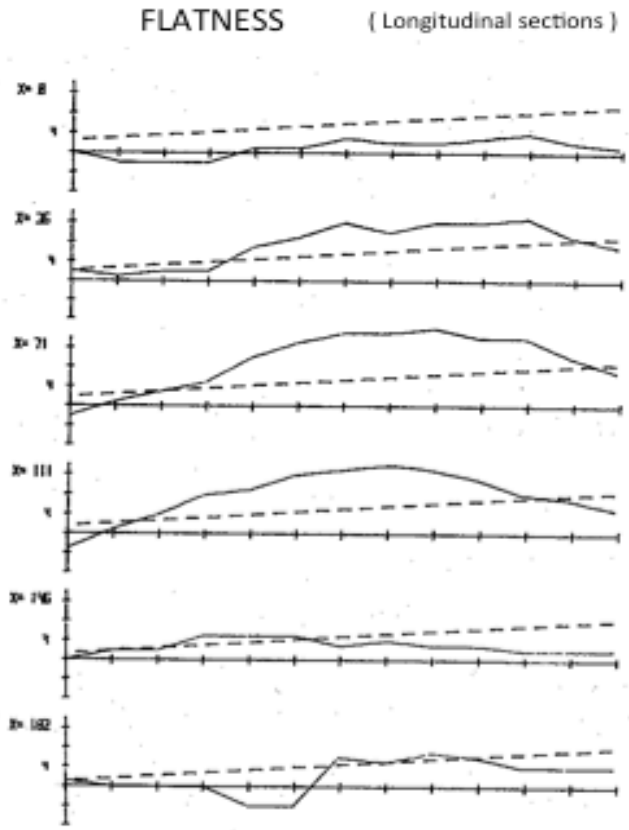

FIGURE 4

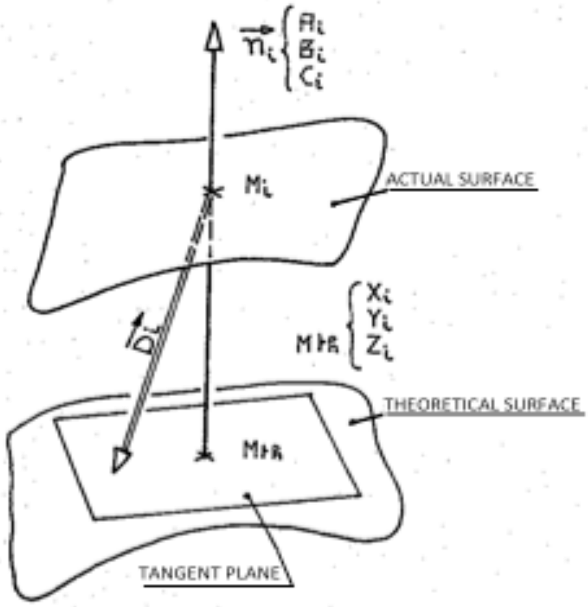

FIG. 3