On the education of the real geometry of mechanical parts through the tolerancing and the tridimensional metrology

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Abstract: The aim of the paper is to present our experience of the education of tolerancing in University. The learning of tolerancing must give means to correctly read and understand standardised specifications, and must show how tolerancing allows to control the real geometry of the part surfaces in order to respect functional requirements. This approach is illustrated through a real case.
Keywords: education of tolerancing**,** geometrical specifications, functional requirements

1 INTRODUCTION

The education of tolerancing is more often limited to the reading of standardised specifications. Generally this learning relies on the numerous examples proposed by the ISO standard [ISO 5459, 1981]. Here, the standpoint is a designer standpoint, which leads to the design of an ideal nominal geometry.

On the other hand, the tridimensional metrology of parts using Coordinate Measuring Machines (CMM) allows to have a view of the real geometry. The real surfaces are known by point sets which are associated to geometrical models referred to *substitute surfaces*. The checking of the specifications is then based on calculations between substitute elements. As a result, such dimensions are not suitable with respect to the ISO standard.

Although the real part geometry is the common point between both types of education (tolerancing and tridimensional metrology), each education is taught independently and following its own point of view.

The paper presents our educational experience in University, which relies on the real geometry of mechanical parts through their standardised description and their tridimensional measurement. Taking the example of the tolerancing between two parallel planes, we show how tridimensional metrology, geometrical specifications and functional requirements are directly linked to the real geometry. The study emphasises the following phenomena:

- the scatter on the association of a geometrical model to a real geometry,
- the limits of tridimensional measurements *vis à vis* the geometrical specifications,
- the relevance of the geometrical specifications *vis à vis* the functional requirements.

Moreover, we propose a conceptual model of tolerancing, issued from ISO standard, which allows reading and interpreting simply all kinds of specifications

2 THE ISSUE

The objective of the education of tolerancing must be double. First, it must give means to correctly read and understand standardised specifications, but moreover, it must show how tolerancing allows to control the real geometry of the part surfaces in order to respect functional requirements.

To solve this issue, the teaching aid we use is the main part of a planing machine. Functional requirements are linked to the expected thickness of the shaving and to the quality of the planed board of wood.

So, the study concerns the determination of the geometrical specifications and the measurement of the studied surfaces in order to check specifications (figure 1). The study consists of two main steps:

- a critical analysis of a usual non-standardised specification,
- the expression of geometrical specifications from the functional requirements of the part.

The critical analysis relies on tests showing the influence of the choice of the measured points on the real geometry for the checking of geometrical specifications. In particular, we study the effect of various point numbers and different point distributions on the association of a geometrical model to a real surface.

In the second step, we propose to define geometrical specifications taking into account the functional requirements of the part and considering the real geometry. The description of the geometrical specifications is performed through a conceptual model of tolerancing.

3 FIRST EXPRESSION OF THE TOLERANCING

3.1 Introduction

The functions of the planing machine are to reduce with a given quantity the thickness of a wood board, and to ensure an acceptable flatness of the planed surface. With the hypothesis that during the rotation the cutting edge of the blade follows a cylinder which is tangent to the plane 2, the distance between the two planes corresponds to the thickness of the shaving. For instance, the thickness is chosen to 0.5mm, and as a result the designer imposes the planes to be parallel, distant of 0.5mm (in the exact position) and with an authorised distance variation of $\pm$0.1mm.

So the first expression of the functions as an ideal geometrical model corresponds to the specification proposed in figure 1.

This specification is obviously non-standardised but it is commonly used by most of the designers.

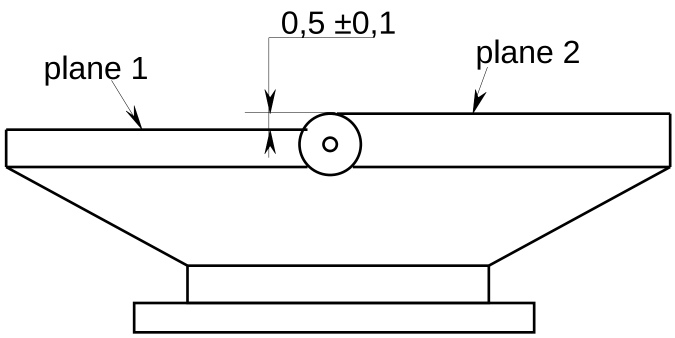

***Figure 1** : First expression of the tolerancing*

3.2 Checking of the specification

The first part of the study concerns the checking of the geometrical specification as indicated on the drawing, even though the specification is non-standardised.
Both specified surfaces of the planing machine are measured using a CMM, following a regular grid (x,y), covering in each case the whole surface. The representation of surface 1 is shown figure 2.

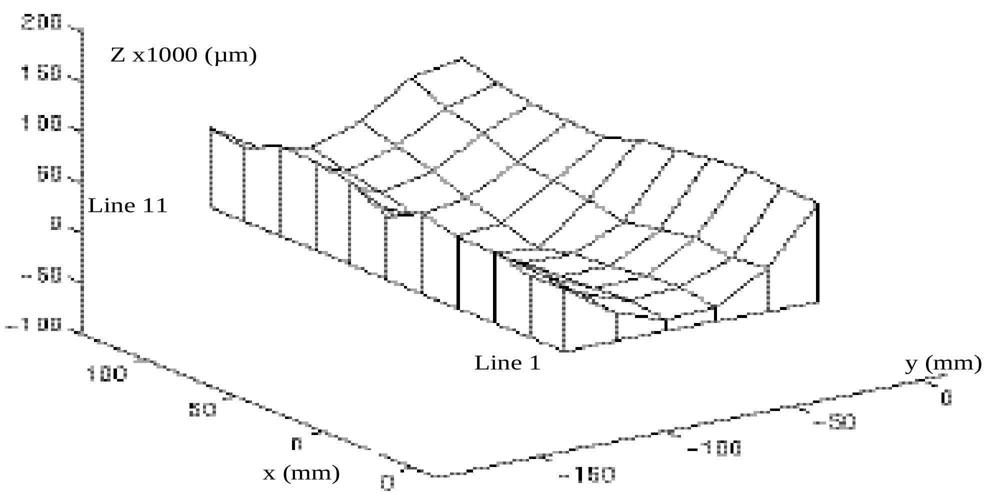

***Figure 2** : The distribution of the measured points*

A point set corresponding to a constant x defines a line (measured in 6 points) and the surface 1 is characterised by 11 point lines and the surface 2 by 12 point lines. Note that for both surfaces, the number of points is relatively high relative to the usual practice.
We can notice that the surface presents a hollow shape, which corresponds to the machining process. In practice, both surfaces were milled with a cylindrical tool for which the rotation axis was not perfectly in coincidence with the normal to the part movement.

As previously exposed, tridimensional metrology which allows the checking of geometrical specifications, relies on the association of geometrical models to point sets. Those models allow to define substitute elements and then, the checking is based on calculations between substitute elements. The association of geometrical models to point sets is performed using the concept of the Small Displacement Torsor [Bourdet et al., 1996]. In order to estimate the influence of the criterion, the association is realised using the Chebyshev criterion (specified in the standard) and using the least-square criterion (commonly used in metrology software).

So, tests will focus on factors influencing the geometrical model association: criterion, number and distribution of the measured points. A didactical software is developed under MATLAB and presents the following practicalities [Bourdet, 1993]:

- visualisation of the real geometry through the measured points,
- suppression of lines of points,
- association of a geometrical model using the Chebyshev criterion,
- association of a geometrical model using the least-square criterion,
- calculation of the distance of all the points to a geometrical model,
- calculation of the distance of the extremity points to a geometrical model.

Let us first consider the problem of the association of a geometrical model to a point set.

3.2.1 Modelling of the surfaces

Each surface is modelled by a plane. The ideal nominal element is a plane which normal is z, and which passes through the lowest point of all the measured points (figure 2). The components of the small displacement torsor correspond then to a small displacement along the z axis(w) and to small rotations around x (α) and around y (β). For each measured point the deviation, e_i, can be expressed by the equation:

$$e_i = \xi_i - (w + \alpha\, y_i - \beta\, x_i) \qquad (1)$$

with $\xi_i = z_i - z_0$

The association problem consists in:

- minimising $\sum_{1}^{n} e_i^2$, using the least-square criterion,
- minimising $[\max(e_i) - \min(e_i)]$, using the Chebyshev criterion.

The resulting form deviation is strongly linked to the used criterion. Moreover, the suppression of lines is also an influencing factor. Tests are performed with various numbers of points for the same distribution (corresponding to the whole coverage of the surface) and with various distributions for the same number of points (the choice of 18 points seems reasonable considering usual measurements of plane surfaces).

In order to emphasise the influencing factors, we consider the displacement of the points A,B,C,D, that materialise the extremities of the plane surface (figure 3). The moved points, during the association, characterise the *substitute plane*. Only the case of plane 1 is presented here, but the same approach can obviously be conducted for plane 2. Table I and table II summarise all the tests of the study. In our tests, the reference is the case including all the lines.

We notice that only the first three series of tests have sense for they correctly represent a large coverage of the plane surface. For the other cases, measured points are located on small portions of the real surface that obviously implies scatters on the form deviation, on the location and the orientation of the substitute element. This provides non-significant results.

Considering the three first tests and even though measured points are correctly distributed, we notice a variation of 30% on the value of the form deviation and a variation of 48% on the location of the substitute plane in function of the number of points, when the Chebyshev criterion is used.

In comparison, results obtained using the least-square criterion present a smaller variation interval for the substitute plane location, only 21%, which corresponds to a diminution of more than 50%.

	Line numbers	form deviation (mm)	upper point (z, in mm)	lower point (z, in mm)	location tolerance (mm)
Chebyshev	all, 1 to 11	0,097	0	0	0
	1-6-11	0,089	0,0011	-0,0192	0,020
	1-11	0,069	0,0144	-0,0322	0,0467
	9-10-11	0,064	0,0146	-0,0662	0,081
	1-2-3	0,073	0,00139	-0,0027	0,041
	1-3-6	0,081	0,0052	-0,0413	0,047
Least-square	all, 1 to 11	0,101	0	0	0
	1-6-11	0,098	0,0045	0,0116	-0,016
	1-11	0,069	0,0048	0,0156	0,02
	9-10-11	0,075	0,068	-0,0408	0,048
	1-2-3	0,074	0,0033	-0,0114	0,014
	1-3-6	0,089	0,0062	0,0318	0,038

Table I

Finally, the least-square criterion, non-standardised but commonly used in metrology software, gives the smallest scatters in the location of the substitute elements.

Results expressed in size of zone	3 first tests		all tests	
	Chebyshev	Least-square	Chebyshev	Least-square
maximum of the form deviation	0,0968	0,1006	0,0968	0,1006
variation on the form deviation	29%	31%	49%	45%
variation on the location of the upper point	0,032	0,016	0,066	0,041
variation on the plane location	0,047	0,020	0,0808	0,048
location error relative to form deviation	48%	21%	83%	49%

Table II

3.2.2 Interpretation of the specification 0,5 ± 0,1

As the associated planes to the surfaces are not parallel, the specification can be interpreted in various ways.

Generally, one of the two planes constitutes the datum. In our study, plane 1 is chosen as the datum and the checking of the specification is performed in two ways.
First, we consider the substitute element of plane 2, characterised by the moved points, A', B', C', D' (figure 3), and we express the distance as the distance of A', B', C', D' to the plane 1. The second way is to calculate the distance of all the measured points to plane 1. Results of the tests are presented in table III.

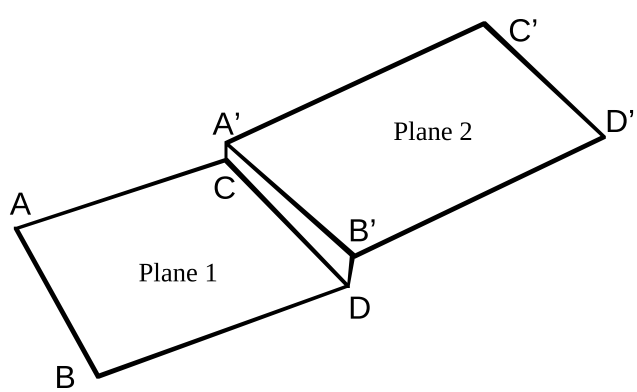

Figure 3 *: Substitute planes*

The various calculation methods of the distance involve results to be quite different, and the choice of the correct method turns out to be difficult.

calculation methods	all lines		lines 1,6,11		lines 1,11	
	dmax	dmin	dmax	dmin	dmax	dmin
distance of A', B', C', D' to plane 1	0,613	0,462				
distance of A', B' to plane 1 :	0,613	0,462				
distance of all the points to plane 1 :	0,666	0,425	0,653	0,425	0,616	0,400

Table III

To conclude this step, all the conducted tests show the obvious difficulty to correctly choose the number and the distribution of points for the checking of geometrical specifications. An a-priori knowledge of the manufacturing process of the part gives necessary information.

The last step of the work is the choice of the most suitable specifications to express the required functions of the part: thickness of the shaving and flatness of the planed surface.

4 EXPRESSION OF THE PART FUNCTIONS IN TERMS OF GEOMETRICAL SPECIFICATIONS

A geometrical tolerance allows to bound the variations of the real geometry into tolerance zones. Considering the ISO standards [ISO 5459, 1981] [ISO 1101, 1983], we have to define:

The real surfaces : corresponding to toleranced features and datum features

The ideal surfaces : corresponding to the datums (single or common), the datum-systems and the associated criteria

The tolerance zone

The tolerance zone is characterised by its shape. Its orientation and its location can be constrained by the datums or the datum systems.

For the example we have to consider the case of two nominally parallel planes. So, we identify :

- two real surfaces, referred as surface 1 and surface 2,
- a datum, for which the geometrical model is*: the outward tangent plane that minimise the form deviation*

Considering the direction of the wood board movement, the plane 1 is chosen as the datum. Surface 2 is thus the toleranced element.

As datum and toleranced element are clearly identified, the geometrical specifications proposed by the standard that can correspond to the functions are (figure 4):

- a specification of form deviation for the datum (flatness, for a plane surface)
- a specification of parallelism of the toleranced element relative to the datum,
- a specification of location of the toleranced element relative to the datum.

For all the geometrical specifications, the shape of the tolerance zones corresponds to two parallel planes.

For the specification of parallelism, the tolerance zone is constrained in orientation: the two parallel planes of the zone are parallel to the datum.

For the specification of location, the tolerance zone is constrained in location: the two parallel planes of the zone are parallel to the datum and located by means of exact dimensions.

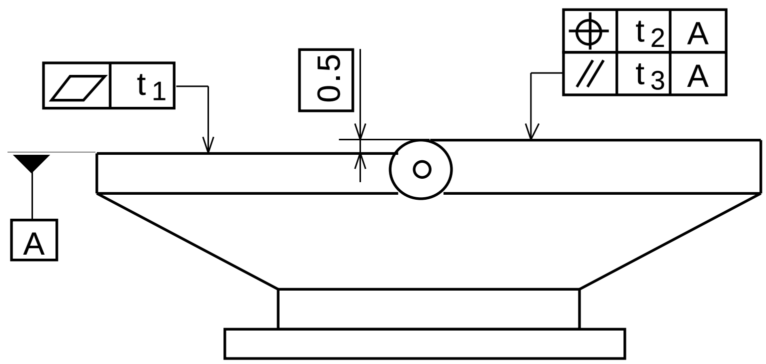

Figure 4 *: Geometrical specifications*

The values of t_1, t_2 and t_3 are defined considering the real geometry of the part and the functional requirements.

4.1 Function : thickness of the shaving, 0,5 ± 0,1

Both planes present a hollow shape, and we can make the hypothesis that the scattering on the association of the geometrical model is of the same order as the form deviation. Taking into account the machining process of the surfaces, the value of the form deviation is estimated to 0.1mm. This value gives a variation of 0.1mm on the thickness of the shaving. So, the dimension of the tolerance zone for the flatness is given by:
t_1 = 0.1mm.
Considering that the blade is tangent to the substitute plane of surface 2, the location of the toleranced element is defined relative to the datum A (associated to the surface 1) with a value of t_2 such as $t_1 + t_2 = 0.2$mm, corresponding to the authorised variation of the shaving thickness. So, the dimension of the tolerance zone for the location is given by:
t_2 = 0.1mm.
If the machining of both surfaces is realised with the same quality, the form deviation is equal to the location deviation. In order to allow the adjustment of the machine tool, we have to reduce the value of the tolerance zone. This imposes the use of a machine tool and of a cylindrical tool of a better quality.

4.2 Function : flatness of the planed surface

The quality of the planed surface is defined through the specification of parallelism. The two surfaces are realised in one machining set up. So, the parallelism deviation is linked to the quality of the machine tool. Considering that the form deviation of surface 2 is of the same order as surface 1, the dimension of the tolerance zone for the location is given by: $t_1 < t_3 < t_2$

And taking into account the previous remarks : $t_1 = 0.05$mm, $t_2 = 0.15$mm, $t_3 = 0.08$mm, this leads to the geometrical specifications presented figure 5.

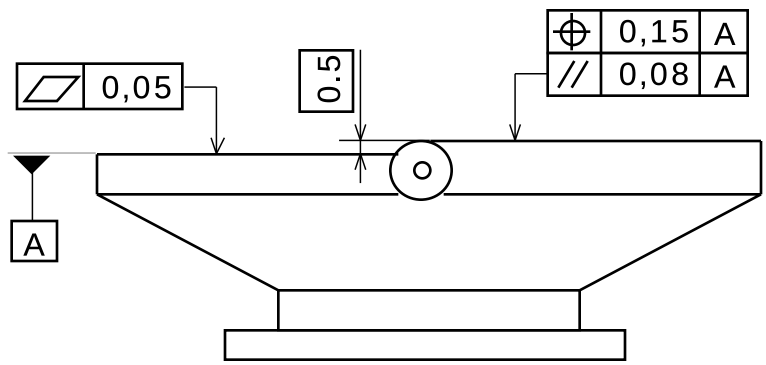

***Figure 5**: Proposal of a tolerancing*

5 CONCLUSION

The teaching of tolerancing, which is generally limited to the learning of the reading of geometrical specifications, must show its link with the real geometry of surfaces and its possibilities to express the functional requirements of parts.

The experimentation we propose to the students consists in the checking of geometrical specifications that formalise a first expression of the functional requirements. Tests lead to bring out the influence of the real geometry of surfaces, in particular for the association of geometrical models to point sets (a plane, in the treated example). We show that, for the same surface, when number and distribution of points vary, the associated model may vary in a zone which size is of the same order as the form deviation of the surface. This effect is more VISIBLE when the association is conducted using the Chebyshev criterion. The use of the least-square criterion allows to bound the size of the incertitude zone, for the zone it is included in the incertitude zone resulting from the association with the Chebyshev criterion.

The numerous possibilities to express the distance between the two substitute planes show that the calculation of the distance must be directly linked to the functional requirements of the two plane surfaces. The use of geometrical specifications issued from the ISO standard show the possibilities to express functional requirements by limiting the variations of the real geometry using tolerance zones.

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