

# Fault-Tolerant Supervisory Control

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IFAC DCDS 2015  
Cancun, Mexico

## 1. Supervisory Control

Discrete-Event Systems

Closed-Loop Configuration

Controller Synthesis

## 2. Fault-Tolerant Control

Naive Approach

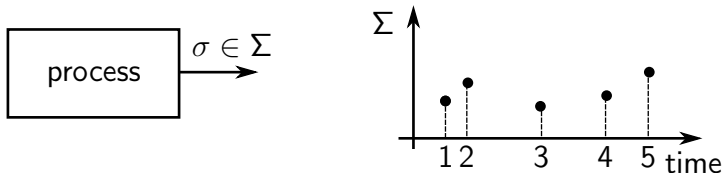
Active Fault-Tolerant Control

Post-Fault Recovery

Fault-Hiding Approach

**A discrete-event system is a model of a process  
... with a particular focus the occurrence of events**

- finite set  $\Sigma$  of symbols  $\sigma \in \Sigma$  (alphabet)
- only event ordering is regarded relevant (logic time)
- within finite time a finite sequence  $s \in \Sigma^*$  is generated
- set  $L \subseteq \Sigma^*$  of sequences that can be generated



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- only event ordering is regarded relevant (logic time)
- within finite time a finite sequence  $s \in \Sigma^*$  is generated
- set  $L \subseteq \Sigma^*$  of sequences that can be generated
- write  $\text{pre } L$  to emphasise that  $L = \text{pre } L$  (local behaviour)

prefix operator  $\text{pre } L := \{s \mid \exists t : st \in L\}$

**A closed language  $\text{pre } L \subseteq \Sigma^*$  is a discrete-event system.**

## Properties

- safety – bad things never happen  
with  $\text{pre } E \subseteq \Sigma^*$ , require  
 $\text{pre } L \subseteq \text{pre } E$
- liveness – good things do happen  
free of deadlocks

$$(\forall s \in \text{pre } L)(\exists \sigma \in \Sigma)[s\sigma \in \text{pre } L]$$

free of livelocks wrt  $M \subseteq \Sigma^*$

$$(\forall s \in \text{pre } L)(\exists t \in \Sigma^*)[st \in M \cap \text{pre } L]$$

For systems with liveness properties:

A natural domain of interpretation for models with liveness properties are  $\omega$ -languages, i.e., sets of inf. strings  $w \in \Sigma^\omega$ .

In the absence of deadlocks, use

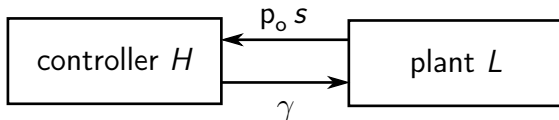
$\lim \text{pre } L := \{w \in \Sigma^\omega \mid \text{pre } w \subseteq \text{pre } L\}$   
to model the process w.r.t. infinite time.

If, in addition, there are no livelocks, choose  $L$  s.t.  $L = M \cap \text{pre } L$  and consider

$\lim L := \{w \in \Sigma^\omega \mid ||(\text{pre } w \cap L)|| = \infty\}$ ,  
to model the process w.r.t. infinite time.

**A language  $L \subseteq \Sigma^*$  is a discrete-event system.**

With the common partitioning  $\Sigma = \Sigma_c \dot{\cup} \Sigma_{uc} = \Sigma_o \dot{\cup} \Sigma_{uo}$  regarding controllable and observable events, consider a plant  $L \subseteq \Sigma^*$ .



- At any time, the controller is provided  $p_o s \in \Sigma_o^*$  where  $s \in \Sigma^*$  is the sequence generated so far;
- in return, the controller applies a control pattern  $\gamma$  of enabled events, where  $\Sigma_{uc} \subseteq \gamma$ ;
- liveness properties of the plant shall be retained.

natural projection  $p_o \Sigma^* \rightarrow \Sigma_o^*$  to remove any symbols not from  $\Sigma_o$ .

**Def.** A controller  $H \subseteq \Sigma^*$  is *admissible w.r.t. the plant*  $L \subseteq \Sigma^*$ , if

$$[H0] \ H = \text{pre } H$$

$$[H1] \ H\Sigma_{uc} \subseteq H,$$

$$[H2] \ H = p_o^{-1} p_o H,$$

$$[H3] \ (\text{pre } L) \cap (\text{pre } H) \text{ does not deadlock, and}$$

$$[H4] \ (\text{pre } L) \cap (\text{pre } H) = \text{pre } (L \cap H).$$

Then,  $K := L \cap H$  represents the closed-loop behaviour. □

**Thm.** Consider the case  $\Sigma_c \subseteq \Sigma_o$ . For a plant  $L \subseteq \Sigma^*$  and an admissible controller  $H \subseteq \Sigma^*$  let  $K = L \cap H$ . Then

[K0]  $K$  is relatively prefix-closed w.r.t.  $L$ ,

[K1]  $K$  is controllable w.r.t.  $L$ ,

[K2]  $K$  prefix-normal w.r.t.  $L$ , and

[K3]  $K$  does not deadlock.

Vice versa, if  $K$  satisfies [K0]-[K3], then there exists an admissible controller  $H$  such that  $K = L \cap H$ .  $\square$



**Control Problem.** Given  $(L, E)$  with plant  $L \subseteq \Sigma^*$  and a specification  $E \subseteq \Sigma^*$  construct an admissible controller  $H \subseteq \Sigma^*$  such that

$$K := L \cap H \subseteq E.$$

**Solution.** All properties are retained under arbitrary union. Thus

$$K^\uparrow = \sup\{K \subseteq L \cap E \mid K \text{ satisfies [K0]–[K3]}\}$$

itself satisfies [K0]–[K3] and is used to extract a minimal restrictive controller.

**Note.**  $E$  can be substituted by a closed language without affecting solutions – it is effectively a safety specification.

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Interpretation by corresponding  $\omega$  – languages

- in general,  $E$  can not be substituted by a closed language
- if  $E$  is (rel.) closed, same solution procedures as with  $*$ -languages (Ramadge 1989, Kumar et al 1993, Moor et al 2012)
- if  $E$  is not (rel.) closed it imposes liveness properties — completely different story
- solution procedure for  $\Sigma_o = \Sigma$  by Thistle and Wonham 1994
- solution procedure for  $\Sigma_o \neq \Sigma$  and closed  $L$  by Thistle and Lamouchi 2009

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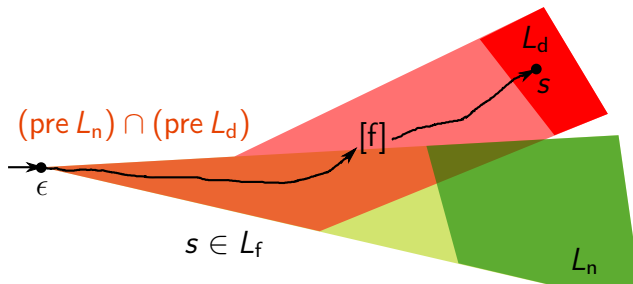
## Fault-Tolerant Control

- a fault is a sudden change of behaviour
  - passive approach: have a single controller that can handle pre-fault and post-fault behaviour (robust control)
  - active approach: detect the fault and switch to another controller (adaptive control)
  - core challenge: switching of plant and controller dynamics
- ... for continuous systems. However, for discrete-event systems ...

**Sudden change of behaviour and switching in the control scheme are the very nature of discrete-event systems. Hence, fault-tolerant control can be synthesised by the same methods as nominal control [??]**

Naive approach to fault-tolerant control:

- nominal plant  $L_n \subseteq \Sigma_n$
- fault event  $f \notin \Sigma_n$ , uncontrollable and unobservable
- degraded post-fault behaviour  $L_d \subseteq \Sigma_f^*$  with  $\Sigma_f = \Sigma_n \dot{\cup} \{f\}$  and  $(\text{pre } L_d) \cap \Sigma_n^* \subseteq \text{pre } L_n$  and  $L_d \cap \Sigma_n^* = \emptyset$



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 $(\text{pre } L_d) \cap \Sigma_n^* \subseteq \text{pre } L_n$  and  $L_d \cap \Sigma_n^* = \emptyset$
- *fault-accommodating model*  $L_f = L_n \cup L_d$  where  
 $\text{pre } L_f = (\text{pre } L_n) \cup (\Sigma_n^* f \Sigma_f^* \cap \text{pre } L_d)$   
 $L_f = L_n \cup (\Sigma_n^* f \Sigma_f^* \cap L_d)$
- likewise, the specification  $E_f = E_n \cup E_d$  to accommodate for degraded post-fault performance  $E_d$

## Naive approach to fault-tolerant control (cnt.)

- invoke synthesis procedure for  $(L_f, E_f)$  to obtain a minimal restrictive admissible fault-tolerant controller  $H_f$ .
- note: diagnosability required only relative to specifications
- option: re-interpretation  $H_f$  as active fault-tolerant control

switch on first escape from  $H_n$ :

$$T := \{s \in \Sigma_f^* \mid \exists \sigma : s\sigma \in H_f \not\rightarrow s\sigma \in p_f^{-1} H_n\}$$

$$H_d := \{s\sigma \in H_f \mid (\text{pre } s) \cap T \neq \emptyset\} \cup \{\epsilon\}$$

$$H_f = ((p_f^{-1} H_n) \cap (\Sigma_f^* - T\Sigma_f\Sigma_f^*)) \cup H_d$$

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- note: in general,  $H_f \cap \Sigma_n^*$  is not admissible w.r.t.  $L_n$

this requires two more closed-loop properties for the synthesis of  $H_f$ :

[K4]  $K \cap \Sigma_n^*$  does not deadlock

[K5]  $\text{pre}(K \cap \Sigma_n^*) = (\text{pre } K) \cap \Sigma_n^*$



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- option: re-interpretation  $H_f$  as active fault-tolerant control
- note: in general,  $H_f \cap \Sigma_n^*$  is not admissible w.r.t.  $L_n$
- option: compute a minimal restrictive nominal controller  $H_n$  that solves  $(L_n, E_n)$  and test whether  $L_f \cap H_f \cap \Sigma_n^* = L_n \cap H_n$

we have  $L_f \cap H_f \cap \Sigma_n^* \subseteq E_f \cap \Sigma_n^* = E_n$   
for free, and, if  $H_f \cap \Sigma_n^*$  is admissible  
w.r.t.  $L_n$ ,

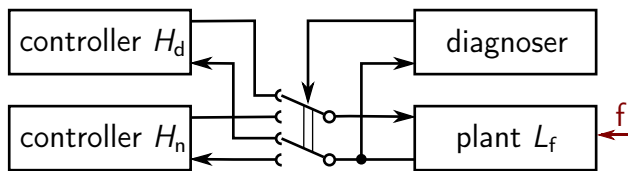
$$L_f \cap H_f \cap \Sigma_n^* \subseteq L_n \cap H_n$$

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- option: explicit diagnosis by controllable event  $F \in \Sigma_n$  with plant  $p_F^{-1} L_f$  and specification  $(p_F^{-1} E_f) \cap \text{pre}(\Sigma_n^* F \Sigma_f^*)$

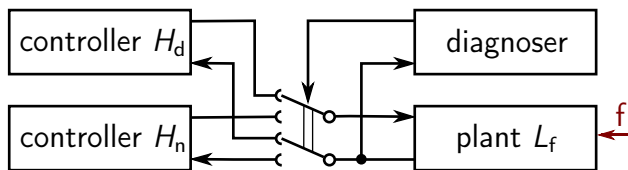
need to interpret  $F$  as forcible event

## Active fault-tolerant control



- require the fault to be diagnosable, denote  $T \subseteq L_d$  the strings corresponding to  $f$ -certain diagnoser states
- require/test that the post-fault behaviour satisfies a safety specification (safe diagnosability)
- design  $H_d$  to take over  $H_n$  when the plant first enters  $T$
- note: nominal pre-fault behaviour is guaranteed
- option: synthesise  $H_d$  online once the fault has been detected

## Active fault-tolerant control



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- note: nominal pre-fault behaviour is preserved
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Diagnosis of DES (Sampath et al 1995)

*diagnoser*: observer automaton with dedicated state labels

*f-certain state*: state in which the fault must have occurred.

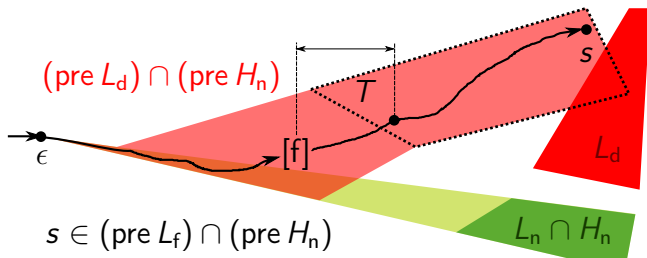
*diagnosability*: require the plant to evolve to an  $f$ -certain state after a bounded number of transitions.

- diagnosability ensures that every string after  $f$  evolves into  $T$

$$T = \{s \in (\text{pre } L_d) \cap (\text{pre } H_n) \mid (p_o^{-1} p_o s) \cap (\text{pre } L_f) \subseteq \Sigma_n^* f \Sigma_f^* \}$$

- safe diagnosability ensures that

$$[(\text{pre } L_d) \cap (\text{pre } H_n)] - T \Sigma_f^* \subseteq E_d$$



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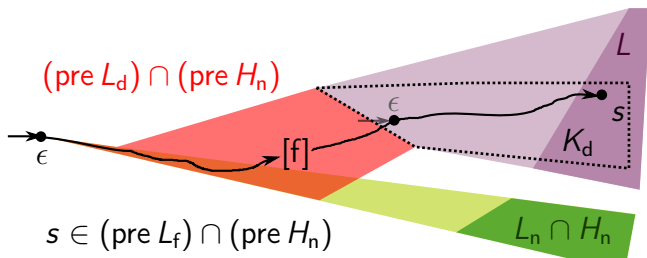
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- synthesise  $H_d$  for the post-fault detection plant

$$L = \{s \mid \exists t : ts \in L_d \mid (\text{pre } s) \cap T = s \in H_n\}$$



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$$L = \{s \mid \exists t : ts \in L_d \mid (\text{pre } s) \cap T = s \in H_n\}$$

- re-interpret within naive approach:

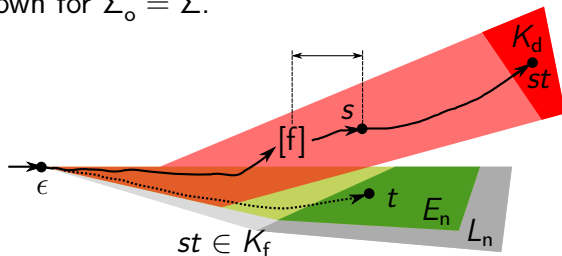
- synthesise  $H_f$  with  $\text{pre } \{s\sigma \in T \mid s \notin T\} \subseteq E_d$
- test for  $L_f \cap H_f \cap \Sigma_n^* = L_n \cap H_n$
- extract  $H_d$  from  $H_f$
- mimic re-initialisation

## Post-Fault Recovery: nominal safety specification

- add a formal closed-loop requirement to [K0]-[K3] for the synthesis of  $H_f$

$$[K6] \quad E_n \Leftarrow K_f / (\Sigma_n^* f)$$

- although [K6] is not retained under union, synthesis procedures are known for  $\Sigma_o = \Sigma$ .



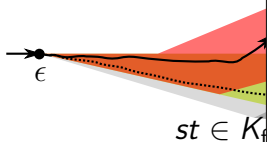


## Post-Fault Recovery: nominal safety specification

- add a formal closed-loop re-synthesis of  $H_f$

$$[K6] \quad E_n \Leftarrow K_f / (\Sigma_n^* f)$$

- although [K6] is not retained, it is known for  $\Sigma_o = \Sigma$ .



Language Convergence (Kumar et al 1993)

$K$  is said to *finitely converge* to  $E$  if there exists a uniform bound  $k$  such that every  $s \in K$  can be decomposed

$$s = vw, w \in E, \text{ and } |v| \leq k.$$

This is written  $E \Leftarrow K$ .

Without the uniform bound, the condition becomes equivalent to

$$K \subseteq \Sigma^* E,$$

which is quite weak.

Alternative approach: refer to the respective  $\omega$ -languages and require

$$\lim K \subseteq \lim(\Sigma^* E)$$

for not-uniform bounded convergence.

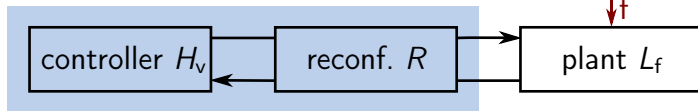


## Post-Fault Recovery: liveness

- a closed loop  $K_f = K_n \cup K_d$  is *fault toleant* if
  - [K7] there exists a uniform bound  $k$  such that for every  $s, t$ ,  $|t| \geq k$  with
$$s \in (\text{pre } K_f) - (\text{pre } K_n) \text{ and } st \in \text{pre } K_f$$
there exists  $u \in \text{pre } K_n, v \in \text{pre } t, |v| \leq k$  with
$$K_f/sv = K_f/v$$
- synthesis problem: given  $L_f = L_n \cup L_d$  and  $E_f$ , compute an admissible controller  $H_f$  such that the closed loop satisfies [K7].
- the property is not retained under union; synthesis procedure exists for  $\Sigma_o = \Sigma$

## Fault Hiding

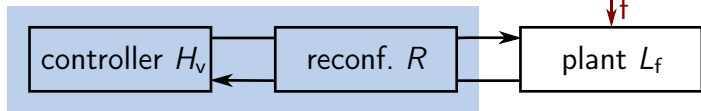
Given  $L_f = L_n \cup L_d$ ,  $E_f = E_n \cup E_d$ , and a solution  $H_n$  to  $(L_n, E_n)$   
fault-tolerant ctrl.



- disconnect nominal controller, i.e.,  $H_v = h(H_n) \subseteq \Sigma_v^*$  with  $\Sigma_v \cap \Sigma_f = \emptyset$ ,  $h$  bijective and applied per event.
- synthesise reconfiguration dynamics  $R \subseteq (\Sigma_v \cup \Sigma_o)^*$  to re-connect
- do so by interpreting  $H_v \parallel L_f$  as plant and use std. procedures on adapted language inclusion specification

## Fault Hiding

Given  $L_f = L_n \cup L_d$ ,  $E_f = E_n \cup E_d$ , and a solution  $H_n$  to  $(L_n, E_n)$   
 fault-tolerant ctrl.



- when using a minimal restrictive solution  $H_v^\uparrow$  for the design, and if the closed loop  $K$  satisfies in addition to [K0]-[K3]

$$[K8] \quad ( \forall s \in \text{pre } K ) [ ((p_v s) \cap h(\Sigma_{uc}) \cap (\text{pre } h(L_n)) \neq \emptyset \\ \Rightarrow s(\Sigma - h(\Sigma_c))^* h(\Sigma_{uc}) \cap (\text{pre } K) \neq \emptyset ]$$

then  $R$  is admissible to any nominal controller that solves  $(L_n, E_n)$ .

- [K8] is retained under union, synthesis procedures are known.
- note: nominal controller does not need to be known

## Summary

Fault-tolerant supervisory control is addressed by the recent literature in various ways, including passive and active approaches, post-fault recovery and fault-hiding.

## Conclusions

- switching is addressed by the common modelling framework — any method for fault-tolerant supervisory control should be interpretable within this framework
- additional features of individual approaches amount to additional closed-loop properties — and novel synthesis problems
- insisting in uniform bounds for diagnosability and language convergence may be too strict for particular applications — discussion in terms of  $\omega$ -languages may turn out beneficial

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**[more references are given in the paper]**

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[more references are given in the paper]